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COMPUTER-AIDED MATHEMATICAL ANALYSIS
OF PROBABILITY OF INTERCEPT FOR
GROUND-BASED COMMUNICATION INTERCEPT
SYSTEM

by

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September 1989

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The probability of detection is a measure of the receiver's capability to detect a signal in the presence of noise. The probability of coincidence is the probability that an intercept system is available, actively listening in the proper frequency band, in the right direction and at the same time that the signal is received.

We investigate the behavior of the POI with respect to the observation time, the separation distance, antenna elevations, the frequency of the signal, and the receiver bandwidths. We observe that the coincidence characteristic between the receiver scanning parameters and the signal parameters is the key factor to determine the time to obtain a given POI This model can be used to find the optimal parameter combination to maximize the POI in a given scenario. We expand this model to a multiple system.

This analysis is conducted on a personal computer to provide the portability. The model is also flexible and can be easily implemented under different situations.

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by

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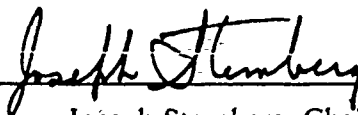
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ABSTRACT

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I. INTRODUCTION

In modern warfare, Electronic Warfare (EW) plays an important role in overall military strategy which concentrates on the neutralization of the enemy's command, control and communications, also called C^3, while maintaining the capability of operating friendly C^3 systems. EW, as defined in a dictionary of military terms generated by the US Joint Chiefs of Staff, is "a military action involving the use of electromagnetic energy to determine, exploit, reduce, or prevent hostile use of electromagnetic spectrum and action which retains friendly use of electromagnetic spectrum."[Ref. 1]

EW is organized into three major categories - electronic warfare support measures (ESM), electronic countermeasures (ECM) and electronic counter-countermeasures (ECCM). Among these three areas, ESM provides a source of information required to conduct the other areas. By definition, ESM is the division of EW involving actions taken to search for, intercept, locate, and immediately identify sources of electromagnetic energy for the purpose of immediate threat recognition and the tactical employment of forces. [Ref. 2] The key functions of ESM are intercepting, identifying, analyzing, and locating sources of hostile radiation. ESM performs the following tasks:[Ref. 3]

- search in time, space and frequency to detect signal activity
- determine signal classification and extract signal intelligence
- determine emitter locations
- decide which actions should be taken (for example, cease minitoring, continue to monitor, apply ECM, and so on).

In other words, ESM is designed to answer several questions related to enemy systems as follows:

- what is it ?
- where is it ?
- what is it doing ?
- what is it going to do ?
- what should be done about it ?

The command and control of forces requires the use of communications. The communication signals may also be intercepted and analyzed to determine the identity,

disposition and intentions of forces. This type of activity is called signal intelligence (SIGINT) which is performed for intelligence gathering. SIGINT is defined as the product resulting from the collection, evaluation analysis, integration, and interpretation of information derived from intercepted electromagnetic emissions. The subdivisions of SIGINT are electronic intelligence (ELINT), communication intelligence (COMINT) and radiation intelligence (RINT). [Ref. 2] ELINT is the intelligence information that is the product of collection and processing, for subsequent intelligence purposes, of potentially hostile, non-communications electromagnetic radiations which emanate from other than nuclear detonations and radioactive sources. COMINT is the intelligence derived from potentially hostile communications by other than the intended recipients. A third division of SIGINT called RINT is the intelligence derived from potentially hostile communications and weapons system by virtue of their unintended spurious emissions, even when in a non-transmitting mode of operation. In military field manuals, foreign instrumentation signals intelligence (FISINT) is taken into consideration instead of RINT. FISINT is the the technical information derived from intercept of electromagnetic emissions, such as telemetry, associated with the testing and operational deployment of foreign aerospace surface and subsurface instrumentation.[Ref. 4]

As a subdivision of SIGINT, COMINT is a strategically oriented activity while radar ESM has a more tactical orientation. COMINT generally focus on producing intelligence data which is not as time critical as radar ESM data. However, there is some overlap between COMINT and the radar ESM in practice. The difference between the radar ESM and COMINT can be explained as follows.

For the radar ESM

- Transmitter and receiver usually collocated
- Two-way range for transmission
- No encryption for message security
- Easier to spoof

For COMINT

- Transmitter and receiver at different locations
- One way range for transmission
- Encryption for message security
- Difficult to creat false message

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An intercept system, in conducting the COMINT operations, usually consists of an antenna, a receiver, a signal display(spectrum analyzer) and an operator. If a signal of interest is transmitted and subsequently acquired by an intercept system, then we consider that the signal has been intercepted. In general, no one has prior knowledge that the signal will be transmitted at a given time, or even the frequency or geometric location of the emitter. Thus, it is generally necessary to conduct a temporal, spectral and spatial search in order to intercept the signal. The environment from which the signal of interest must be extracted normally contains many signals which are of no interest at all. The various signals appear and disappear, creating a dynamic environment which must be continually examined if the signals of interest are to be intercepted.

In signal intercept systems, the ability of the system to perform its function is directly related to the probability that the signals of interest will be received, detected and identified. This is referred to as the Probability of Intercept (POI). In an ideal system, POI should be unity. The POI concept can be applied to the communication environment to analyze the performance of communications intercept sytems. The POI can be written as a function of an observation time(t), a distance between the emitter and the intercept station(d), a frequency of the signal (f) and the various interceptor parameters. Some of the parameters are not controllable by the intercept station. These parameters must be specified in a given scenario. The other parameters can be selected to maximize the POI in the COMINT operation. Not many references are available in the POI applications to the COMINT operation while many are available in the radar ESM system.

The objective of this thesis is to analyze the effects of the intercept system parameters to the POI in the COMINT operation. However, because of the complexities of the signal environment and the COMINT receiver, we have to simplify the scenario in order to analyze the POI mathematically in the COMINT operation. The scenario considered in this thesis is that a ground mobile intercept station (GMIS) is deployed in a forward area to intercept the short range tactical communication of the hostile emitter. The accuracy of this analysis is limited by the degree of simplification of the scenario.

In Chapter Two, we specify the COMINT scenario and review some factors affecting the POI. In Chapter Three, we define the problem to be analyzed and present the fundamental theory for deriving the POI, the probability of detection (which is a function of signal-to-noise ratio), and the probability of coincidence for temporal, spectral and spatial coincidence factors (which can be described as window functions). Furthermore, we discuss the POI for multiple systems In Chapter Four, we explain the method of analysis and analyze the relationship between the POI and the various parameters,

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using the results obtained by MATHCAD, a computer software package for solving mathematical equations. In Chapter Five we conclude the discussion and make recommendations for further investigation.

II. BACKGROUND

A. COMMUNICATION INTERCEPT SCENARIO

Military radio communication equipment generally operates in the high frequency (HF), very high frequency (VHF) and ultra-high frequency (UHF) portions of the electromagnetic spectrum. The operation frequencies are 2 to 30 MHz for HF, 30 to 88 MHz and 116 to 150 MHz for VHF and 225 to 400 MHz for UHF. The VHF/UHF bands are used for line-of-sight (LOS) communication while the HF band is used for both longer range over-the-horizon (OTH) transmissions using sky waves and shorter range communication using ground waves.[Ref 2] Appendix A shows some of the parameter characteristics of the potential enemy emitters.

The volume of communication signals can be very large with 9000 channels potentially available at HF, 3680 at VHF, and 7000 at UHF.[Ref. 2] In addition, a large volume of military communication is transmitted by telephonic and telegraphic means over either wire or radio relay links. COMINT generally does not focus at these latter types of communications systems, since the wire communications require too much effort to make intercept possible.

The function of COMINT receivers which operate against communications systems closely parallels the use of ELINT receivers against non-communication emitters. COMINT is used to build up a library of the characteristics of enemy communication emitters. This database is then used in battlefield situations along with communication ESM receivers.

There are four primary functions performed by communication ESM systems, which are; identification of the operating frequency of active emitters, measurement of their bearing or location, analysis of traffic to assess its threat significance and maintenance and updating of the current database. The first two functions are performed by the spectrum analysis and the Direction Finding (DF) equipment. DF is a key element in sorting and locating communication signals due to the dense communication signal environment.

A large number of both AM and FM communication signals transmitted by low-power mobile and high-power fixed stations, at various locations, causes the dynamic range required at a typical intercept site to equal 80 dB. [Ref. 2] The exceptionally long propagation paths possible at HF generally cause a large percentage of channel

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occupancy in this band. In LOS VHF/UHF communication systems, with typically 25 KHz channels, the occupancy is expected to be somewhat less than that in the HF band. These high occupancies, and the wide dynamic range, require the use of a high sensitivity receiver with typically 100 dB suppression of signals in adjacent channels.

Communication ESM receivers must be sensitive, accurate, invulnerable to large out-of-channel interfering signals and remotely controlled. The frequency coverage extends from 2 to 500 MHz, where the lower band (HF) consists of both long-range sky wave and short-range ground wave transmissions, and the upper band (VHF/UHF) is used for short-range vehicle and man-pack communications.

Intercept receivers which look for short-range emitters must be stationed in forward areas, and therefore must be mobile and rugged. The requirements for communication signal interception are summarized in Table 1.[Ref. 5]

Table 1. REQUIREMENTS FOR COMMUNICATIONS INTERCEPTION

Frequency Range (MHz)	2 - 500
Receiver Accuracy (Hz)	100
Signal Type	1 ms duration to modulated CW
Sensitivity (dBm)	better than -105
Resolution (KHz)	(HF) 1 (VHF/UHF) 5 -25
Instantaneous Dynamic Range (dB)	greater than 80
Amplitude Accuracy (dB)	1
Bearing Accuracy (degrees)	1
Signal Density	10^3 - 10^4 emitters
Intercept Probability (percent)	100

The communication ESM receivers typically feed into a command center where the various interceptions are analyzed and decisions are made to employ countermeasure techniques against high priority communication links.

B. SOME FACTORS AFFECTING INTERCEPT PROBABILITY

1. Space Attenuation Factor - Received Signal Power

The primary methods of communications by the various elements in infantry or mechanized divisions are typically frequency modulated (FM) radios and amplitude modulated (AM) radios. FM utilizes voice transmission for short distance LOS

communications and AM uses digital format type transmission for long distance communications.

General tactical AM/FM radios operate in the frequency range of 2 to 500 MHz with typical output power of from 1 watt (in portable manpacked radios) to 30 watts or more for the vehicular mounted radios. A detailed description of the emitters of interest is given in Appendix A.

The interception may occur if the sensitivity of the receiver is appropriate to the transmitter output power. In case of propagation above 30 MHz, free space propagation is assumed if a LOS path exists. In this case, the power relationship between the transmitted and the received can be expressed as follows.[Ref. 6: P.1124]

$$S_i = \frac{P_t G_t G_r g}{L_p}$$

where

S_i = Available signal power at the receiver input in milliwatts

P_t = Power radiated from the transmitting antenna in milliwatts

G_t = Power gain due to directivity of the transmitter antenna

G_r = Power gain due to directivity of the receiver antenna

g = Multipath factor

L_p = Propagation (or Path) loss

Available signal power at the receiver input in decibels is:

$$S_i(dBm) = P_t(dBm) + G_t(dB) + G_r(dB) + 10 \log g - L_p(dB)$$

where

$S_i(dBm)$ = Available signal power at the receiver input in decibels below one milliwatt

$P_t(dBm)$ = Power radiated from transmitted antenna in decibels below one milliwatt

$G_t(dB)$ = Power gain due to directivity of the transmitter antenna in decibels

$G_r(dB)$ = Power gain due to directivity of the receiver antenna in decibels

$L_p(dB)$ = Propagation loss in decibels

The effective transmitted power, $P_t G_t$, is in the range of 0.5 to 50 watts for the scenario under consideration. A minimum of four essential parameters must be supplied in order to calculate the propagation loss. These are the carrier frequency (f) in

megahertz, the path distance (*d*) in kilometers and the transmitting and receiving antenna height above ground (*h*, and *h*,) in meters. Other path parameters used in the computations such as horizon distances and elevation angles, may be derived from these values and available terrain-information.

The free-space basic transmission loss is

$$Lp_{fs}(dB) = 32.45 + 20 \log f(MHz) + 20 \log d(km)$$

where

$Lp_{fs}(dB)$ = Free space path loss in decibels

$f(MHz)$ = Center frequency of the signal in megahertz

$d(km)$ = Distance between emtter and intercept syster in kilometers

For LOS calculations for radio signals, this equation provides a good approximation as long as the assumption of homogeneous atmosphere is made and first Fresnel zone clearance is achieved. The description of full and incomplete first Fresnel zone clearance is shown in Fig.1. The method of calculation of first Fresnel zone clearance is well expressed by Jordan [Ref. 7: p.33-17].

Since most FM tactical radios are normally stationed close to the ground, or grazing LOS as depicted in Fig.1(b), first Fresnel zone clearance is assumed to be incomplete for most transmissions and an additional six decibel loss is assumed, as shown in Fig.2.

The additional-attenuation factor should be computed using methods based on different propagation mechanisms. Well within radio LOS, the formulas of two-ray optics are used to compute attenuation relative to free space. Just beyond LOS, diffraction is the dominant mechanism. At great distances, well beyond the radio horizon, the dominant propagation mechanism is usually forward scatter.[Ref. 8] The detailed description of these propagation mechanisms is beyond the scope of this thesis. Here, the more practical concern is for short range (LOS transmission) and long range (beyond radio horizon) transmission.

When the radio LOS does not exist, path loss is more severe than described above. Soil composition, horizontal distance, location and height of obstacles, antenna heights relative to curvature of the earth and atmospheric conditions tend to alter the attenuation drastically. In this case, the following equation for path loss, which takes

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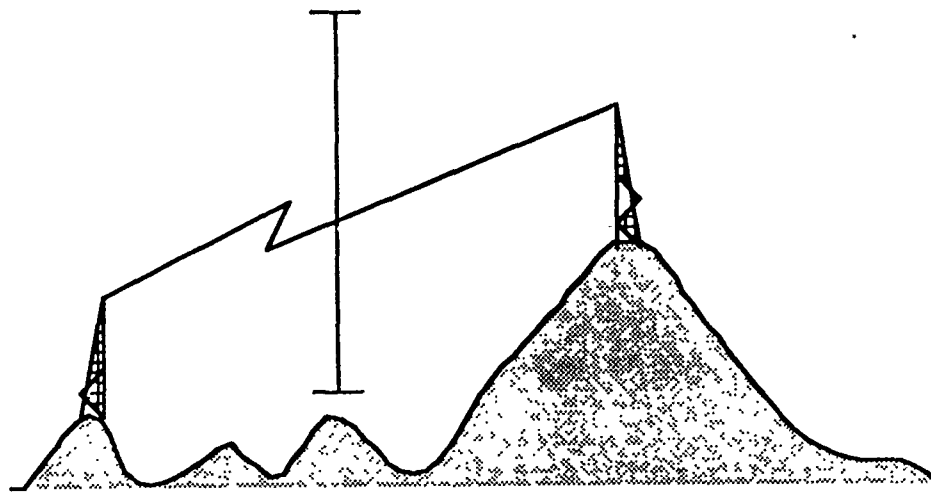
(a) Full first Fresnel zone clearance
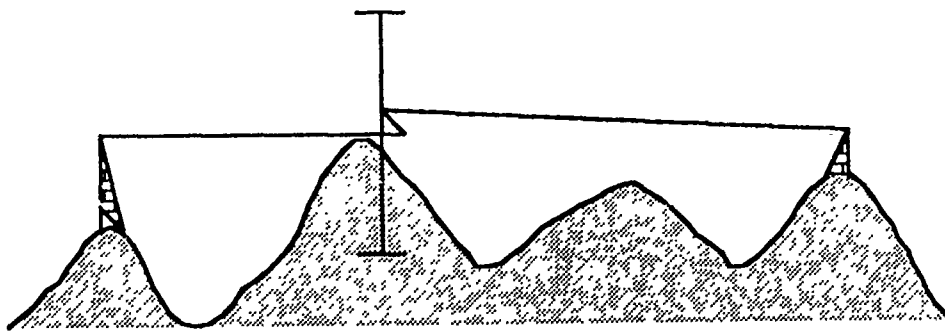
(b) Incomplete first Fresnel zone clearance

Figure 1.   Full and incomplete Fresnel zone clearance
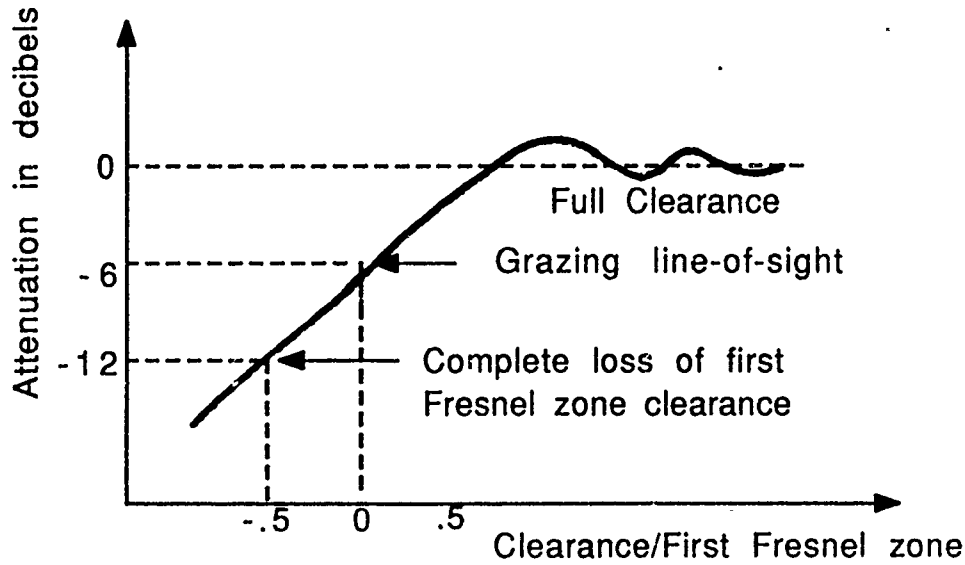
Figure 2. Loss due to lack of first Fresnel zone clearance

into account the lack of first Fresnel zone clearance, terrain loss and space loss, applies.[Ref. 9]

$$Lp_{sp}(dB) = 108 + 20 \log f(MHz) + 40 \log d(km) - 20 \log h_t(m)h_r(m) + 12$$

where

    $h_t(m)$ = Transmitter antenna height in meters

    $h_r(m)$ = Receiver antenna height in meters

    $f(MHz)$ = Center frequency of the signal in megahertz

    $d(km)$ = Distance in kilometers

Directivity and gain are measures of how well energy is concentrated in a given direction. Directivity, or power gain, is the ratio of power density in that direction to the power density that would be produced if the power were radiated isotropically. This ratio is equal to that of the effective area of the antenna to the effective area of an isotropic antenna. The characteristics of antenna parameters are shown in Table 2.

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Table 2. THE CHARACTERISTICS OF ANTENNA PARAMETERS

Type	Gain	Effective Length	Effective Area	3-dB Beamwidth
Isotropic	1		$\lambda^2/4\pi = 0.0796\lambda^2$	$.360°$
Short Dipole	1.5	h	$3\lambda^2/8\pi = 0.1194\lambda^2$	$90°$
$\lambda/2$ Dipole	1.6409	λ/π	$30\lambda^2/\pi R_r = 0.1306\lambda^2$	$78.078°$
$\lambda/4$ Monopole	3.2818	λ/π	$30\lambda^2/\pi R_r = 0.2612\lambda^2$	$78.078°$
Small Loop	1.5	$\pi N k D^2 \mu_e/4$	$3\lambda^2/8\pi = 0.1194\lambda^2$	$90°$
Parabolic Reflector	$0.54(\pi D/\lambda)^2$	D	$0.54S$	$61\lambda/D°$
Horn	$0.81(\pi D/\lambda)^2$	D	$0.81S$	$50\lambda/D°$

where

h = Antenna height (length) in meters

λ = Wavelength of the signal in meters

D = Aperture diameter in meters

N = Number of turns

k = $2\pi/\lambda$

μ_e = Effective permeability

S = Aperture area

Here, for the purpose of the study, it is assumed that the transmitter uses a omnidirectional isotropic antenna and the receiver uses a parabolic reflector. The apparent power gain due to a parabolic reflector is given by:

$$G_r(dB) = 0.54\left(\frac{\pi D}{\lambda}\right)^2$$

Multipath effect should not be negligible. This effect is very important to the accuracy of the probability of detection calculation since it varies according to the combination of the distance between two antennas and the center frequency of the signal. When there is one reflected ray combining with the direct ray at the receiving point as shown in Fig.3. the resulting field strength is related to the free space intensity, irrespective of the polarization, by

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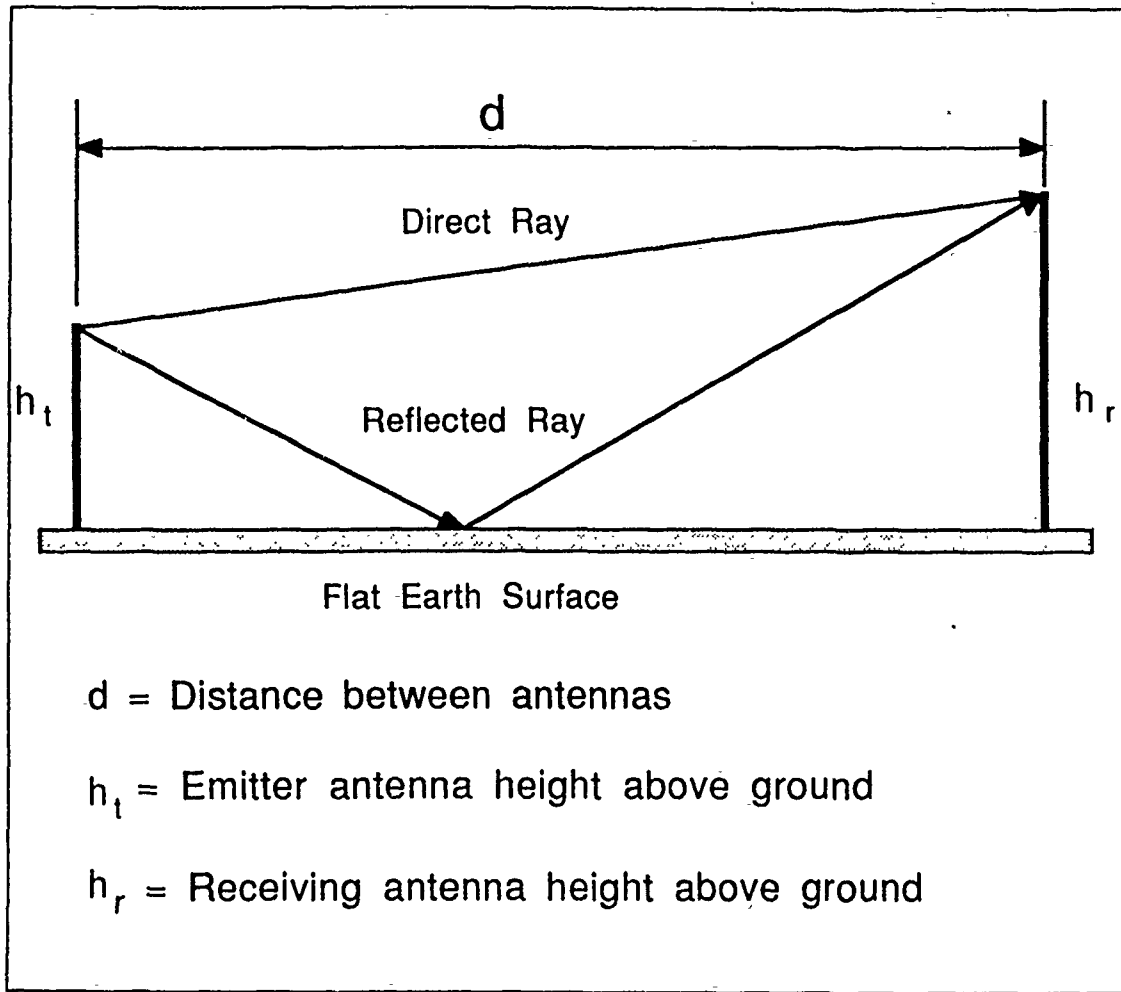
Figure 3. Interference between direct and reflected rays

$$E_r = 2E_d \left| \sin 2\pi \left(\frac{\delta}{2\lambda} \right) \right|$$

where

      E_r = Resulting field strength

      E_d = Direct ray field strength

      δ = Geometrical length difference between direct and refractive paths

        $(\approx 2h_t h_r / d)$

where h_t and h_r are the heights of the antennas in meters above the reflecting plane tangent to the effective earth. The ratio of the reflective and the direct ray field strength

can be written as a multipath factor, g, which the range is $0 \le g \le 2$. So the multipath factor can be written as

$$g = \left| \frac{E_r}{E_d} \right|$$

$$= \left| 2 \sin 2\pi \left(\frac{h_t h_r}{\lambda d} \right) \right|$$

2. System Noise Factor - Noise Figure

In the absence of noise, there would be no degradation of signal quality and one would need only gain to overcome propagation losses. Noise can mask weak signals and create uncertainty in others. Random noise arises from several sources, including external radiation, noise generated internally called Johnson or thermal noise, shot noise from vacumm devices, transistor noise and equivalent noise sources such as lossy elements that contribute effective noise power. This random noise is characterized as the wideband with a uniform spectral density and the Gaussian amplitude probability distribution.

Among various types of noises, the noise generated by the receiver is very significant at a very high frequency. For this reason, it is important to review the source of noise in a typical superheterodyne receiver and the methods commonly used to describe this noise.

Generally speaking, a receiving system consists of the antenna, mixer, amplifier and detector, where mixer, local oscillator, amplifier and detector comprise the receiver. The antenna is considered as a device which reflects its radiation resistance at the input of the receiver from a thermal reservoir contained in that portion of space observed by the antenna. If one considers the observed medium to be a composite black body at temperature T ($^\circ K$), the radiation resistance of the antenna will come into equilibrium with the temperature of this reservoir. The power input to the receiver is then Johnson noise power.

When calculating receiver signal-to-noise ratio (SNR), it is common for engineers to use an approximate equation for noise power at the receiver input. The available thermal noise at the receiver input terminal is given by the following equation.

$$N_i = kT_0 B_r$$

where

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N_t = noise power in Watts

k = Boltzmann's constant (= $1.38 \times 10^{-23} J/°K$),

T_0 = standard temperature ($290°K$),

B_r = the equivalent rectangular bandwidth of the receiver in hertz

Expressed in decibels, the following relationship results:

$$N_i(dBm) = -114 + 10 \log B_r(MHz)$$

where $N_i(dBm)$ is noise power represented in decibels below one watt and $B_r(MHz)$ is receiver effective bandwidth in megahertz.

The noise power of a practical receiver is always higher than the thermal noise of an ideal receiver because noise is introduced by every component in the receiver. The noise figure, as given in following relationship, is

$$F_n = \frac{(SNR)_i}{(SNR)_o}$$

and

$$F_n(dB) = 10 \log F_n$$

where

F_n = Noise figure

$(SNR)_i$ = SNR at the input of the receiver

$(SNR)_o$ = SNR at the output of the receiver

$F_n(dB)$ = Noise figure in decibels

Since the input SNR is always greater than the output SNR, the noise figure is always greater than unity.

3. Scanning Factor - Superheterodyne Receiver

A superheterodyne receiver is the most commonly used receiver in communications because of its high sensitivity and selectivity. Almost all commercial radios and radar receivers are of this type. In EW applications, superheterodyne receivers are used to isolate an input signal and measure from its fine-grain information. A superheterodyne receiver uses filtering, a mixer, and a local oscillator to translate the received signal to a lower intermediate frequency (IF). Filtering and amplification that

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would not be possible at the signal frequency, are possible at this lower IF. Because of this, a superheterodyne receiver possesses greater frequency selectivity compared to other types of receivers.

A basic superheterodyne receiver is shown in Fig.4. This receiver is composed of a mixer, a local oscillator (LO), an intermediate-frequency (IF) filter, an IF amplifier and a video detector. The LO generates a continuous-wave (CW) signal of frequency f_{LO}. If the input signal frequency is f_{RF}, the mixer will shift f_{RF} to f_{IF}, which is the difference frequency of f_{LO} and f_{RF}. This procedure is called *down-conversion*. The IF filter following the mixer is a bandpass filter that is used to pass the desired IF signal and to stop all other frequencies generated in the mixer.

The IF filter is also part of the frequency measurement circuit, because f_{LO} and f_{IF} are known, f_{RF} can be measured. The IF amplifier following the IF filter will provide most of the gain of the receiver. This gain will increase the sensitivity of the receiver. Following the IF amplifier is a crystal video detector. The detector is an envelope detector that converts microwave energy to a video signal. The effect of the video bandwidth should be considered. A video amplifier, following the detector, is often used to amplify the video signals for further processing. In an ESM receiver, a comparator or threshold detector is often used after the video amplifier to detect the existence of the input signals. When the input signal is near but below the threshold, the noise riding on the signal may still trigger the comparator.

Because of its narrow input bandwidth, a superheterodyne receiver has the highest sensitivity and dynamic range of all EW receivers. However, the narrow bandwidth will critically limit the POI. To cover a wider input bandwidth, the receiver can be made to scan a given bandwidth at a fast rate, repeatedly. This type of receiver is often referred to as a scanning superheterodyne receiver.

A typical scanning superheterodyne receiver is the narrowband YIG-tuned type. With this receiver, each frequency resolution cell of interest is examined sequentially by tuning the YIG local oscillator. When an activity is detected in any frequency resolution cell, the sweep stops to allow the processor to analyze the detected signals. The RF bandwidth of the narrowband YIG-tuned superheterodyne receiver is limited by the bandwidth of the YIG-tuned preselection filter, typically ranging from 20 to 60 MHz, depending on the number of stages within the filter structure.[Ref. 10]

In scanning the local oscillator, to attempt 100% POI with high sensitivity, if scan time is less than the shortest signal duration; 100% POI is guaranteed. But in this

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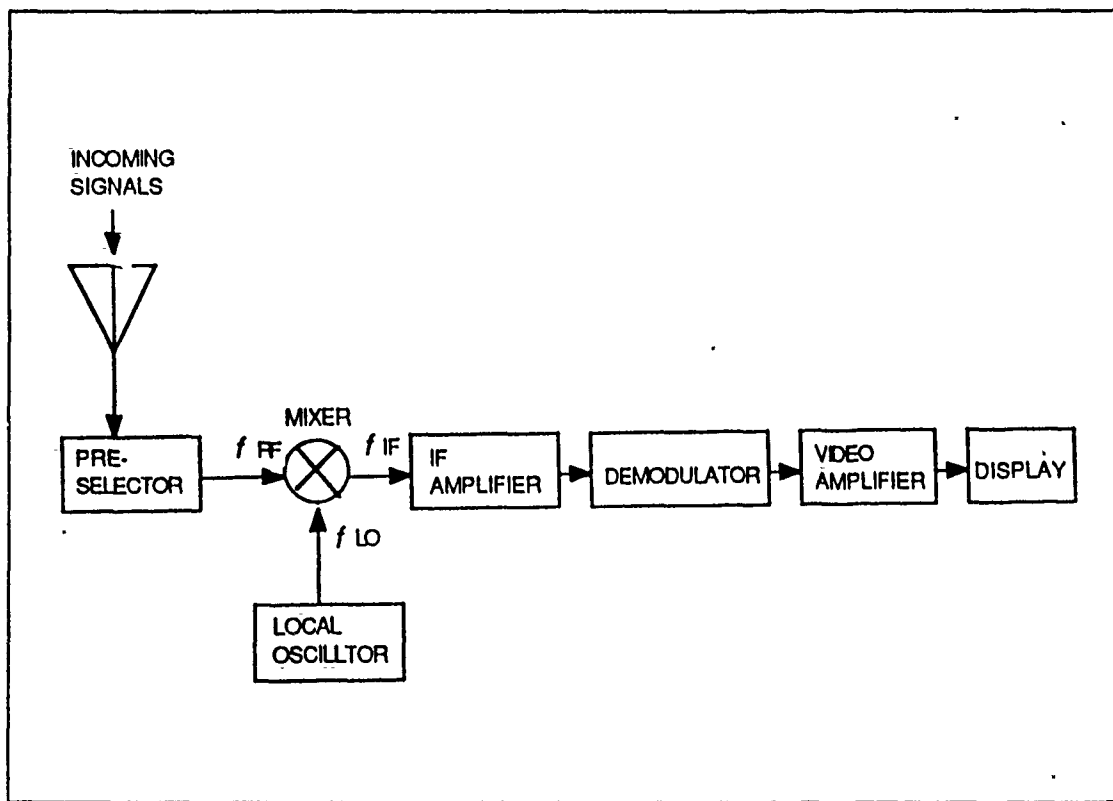
Figure 4. Diagram of a basic superheterodyne receiver

case, if signal is not in IF bandpass long enough to rise to full amplitude, there is a significant loss in sensitivity of the receiver such as:[Ref. 11]

$$L_s(dB) = \left[1 + 0.195\left(\frac{D_s}{T_s B_r^2} \right)^2 \right]^{-1/4}$$

where

$L_s(dB)$ = Scanning loss over fixed signal in decibels

D_s = Total scan width in hertz

T_s = Superhet Scan Time in seconds

B_r = Receiver Acceptance Bandwidth in hertz

Therefore the dwell time, at a given frequency, should be longer than the reciprocal of the IF bandwidth. It is desired to dwell on a given frequency for a sufficient length of time to improve the probability of intercept and to allow time domain

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parameters to be measured by the ESM receiver. Another consideration for the sweep rate is due to the effect of the sweep rate on the amplitude of the envelope at the output of the IF amplifier. If the sweep rate is too fast, the IF output becomes essentially the impulse response, and the amplitude of output is decreased. In order to avoid the attenuation in amplitude, the dwell time should be longer than the signal build-up time. Suppose that the dwell time of the receiver equals to $\dfrac{B_r}{df/dt}$, where df/dt is time on frequency, then the relationships of those parameters follows.

$$\frac{B_r}{df/dt} \geq \frac{1}{B_r}$$

where $\dfrac{df}{dt} = \dfrac{D_s}{T_s}$
or

$$\frac{D_s}{T_s} \leq B_r^2$$

This gives the fastest scan rate as follows:

$$\left(\frac{D_s}{T_s} \right)_{max} = B_r^2$$

That is, we can minimize the scanning loss L_s by $B_r \geq \sqrt{D_s/T_s}$, but should realize that increasing IF bandwidth gives poorer resolution in frequency and thus poorer signal information.

A primary component of a typical communication intercept system is the double or triple-conversion superheterodyne receiver, which is normally designed for operation over the entire HF band, and part of the VHF/UHF bands. A high performance HF receiver uses a 1 Hz step synthesizer which has the memory capability to hold the 100 most significant threat channels. Frequency stability is ± 1 ppm over the temperature range and the single sideband sensitivity is $1 \, \mu V$ for 10 dB output SNR. Dynamic range is 80 to 100 dB for signals spaced at least 20 kHz apart. Single-sideband (SSB), AM,FM, and CW can be individually identified and frequency shift keying (FSK) can be decoded with an individual modem. [Ref. 2]

4. Signal to Noise Ratio

Signal-to-noise ratio (SNR) is the predominant factor in determining probability of detection in the presence of the noise. The SNR is a function of various transmitter and the receiver parameters. The output SNR of the receiver can be expressed as follows.

$$SNR(dB) = S_i(dBm) - N_i(dBm) - L_s(dB) - F_N(dB)$$

where

$SNR(dB)$ = Receiver output SNR in decibels

$S_i(dBm)$ = Received signal power at the input of the receiver
in decibels below one milliwatt

$N_i(dBm)$ = Receiving system noise power in decibels below one milliwatt

$L_s(dB)$ = Receiver scanning loss in decibels

$F_N(dB)$ = Noise figure in decibels

Let us combine all the parameters in one equation. Then the output SNR is:

$$SNR(dB) = P_t(dBm) + G_t(dB) + G_r(dB) - 10 \log B_r$$
$$+ 114 - L_p(dB) - L_s(dB) - F_N(dB)$$

or

$$SNR = 10^{\frac{SNR(dB)}{10}}$$

Since we consider two types of path loss, which are the free space path loss within LOS, and the spread path loss well beyond the radio horizon; the equation becomes:

$$SNR_{fs}(dB) = P_t(dBm) + G_t(dB) + G_r(dB) + 114$$
$$- 10 \log B_r - Lp_{fs}(dB) - L_s(dB) - F_N(dB)$$

or

$$(SNR)_{fs} = 10^{\frac{SNR_{fs}(dB)}{10}}$$

also

$$SNR_{sp}(dB) = P_t(dBm) + G_t(dB) + G_r(dB) + 114$$
$$- 10 \log B_r - Lp_{sp}(dB) - L_s(dB) - F_N(dB)$$

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or

$$SNR_{sp} = 10^{\frac{SNR_{tp}(dB)}{10}}$$

The probability of intercept is a function of the sensitivity of the receiver, and the receiver sensitivity is strongly related with this SNR.

5. Geometric Consideration - Radio Horizon

Under normal propagation conditions, the refractive index of the atmosphere decreases with height so that radio rays travel more slowly near the ground than higher altitudes. This variation in velocity with height results in bending of the radio rays.

Uniform bending may be represented by straight line propagation, but with the radius of the earth modified so that relative curvature between the beam path and the earth remains unchanged. The new radius of the earth is known as the effective earth radius, and the ratio of the effective earth radius to true earth radius is usually denoted by K. The average value of K in temperate climates is about 1.33; however, values from about 0.6 to 5.0 are to be expected.

Under certain atmospheric conditions, the refractive index may increase with height, causing the radio ways to bend upward. Such inverse bending results in a increase in path clearance on LOS paths, but a decrease in reception.

The distance to the radio horizon over smooth earth, when the height, h, is very small compared with the radius of the earth, is given with a good approximation by the expression[Ref. 8]

$$d = \sqrt{0.002KRh}$$

where

d = the smooth earth horizon distance in kilometers

K = ratio of the effective to the true radius of the earth

R = the true earth's radius in kilometers

h = the effective antenna height in meters

Assuming $K = 1.33$, replacing for the value of R and K in the above equation, then

$$d = 4.12\sqrt{h}$$

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The radar horizon between the transmitter antenna and the receiver antenna in kilometers is

$$R_{HZ} = 4.12(\sqrt{h_t} + \sqrt{h_r})$$

The graphical description of this scenario is shown in Fig.5.

This relation shows that once the altitudes are given, if the slant range between the emitter and the intercept system exceeds R_{HZ}, then no signal from the transmitter could be detected by the intercept system.

The SNR is then modified to take into account the radio horizon as follows

$$SNR = \Phi(R_{HZ} - R_{TR}) \times SNR_{fs} + \Phi(R_{TR} - R_{HZ}) \times SNR_{sp}$$

or

$$SNR = \begin{cases} SNR_{fs} & R_{HZ} \geq R_{TR} \\ SNR_{sp} & R_{HZ} < R_{TR} \end{cases}$$

where

R_{TR} = Distance between the transmitter and the receiver in kilometers

R_{HZ} = Radio horizon distance in kilometers

note that $\Phi(x)$ is called Heaviside step function, which returns 0 when the argument is less than 0, otherwise returns 1. That is, if the distance between the transmitter and the intercept system is within the radio horizon, then SNR is equal to SNR_{fs}, otherwise SNR is equal to SNR_{sp}.

6. Receiver Sensitivity - ESM Line

One basic requirement for interception of signal is that some portion of the electromagnetic energy radiated from the emitter should be impinged on the ESM antenna. In order to visualize the SNR for the case under consideration, we can make use of ESM line as shown in Figure 6. This figure shows the relationship between the receiver acceptance bandwidth and the maximum allowable range. The slope of the signal power line is constant and equals 20 dB/decade, showing the reciprocal of R^2 dependence of the signal power with range, as expressed earlier. The noise power line is horizontal, showing the independence of the noise power with range.

The interception of this signal is determined by the relationship of the signal power (which is the function of the peak power of emitter, the gains of both antennas,
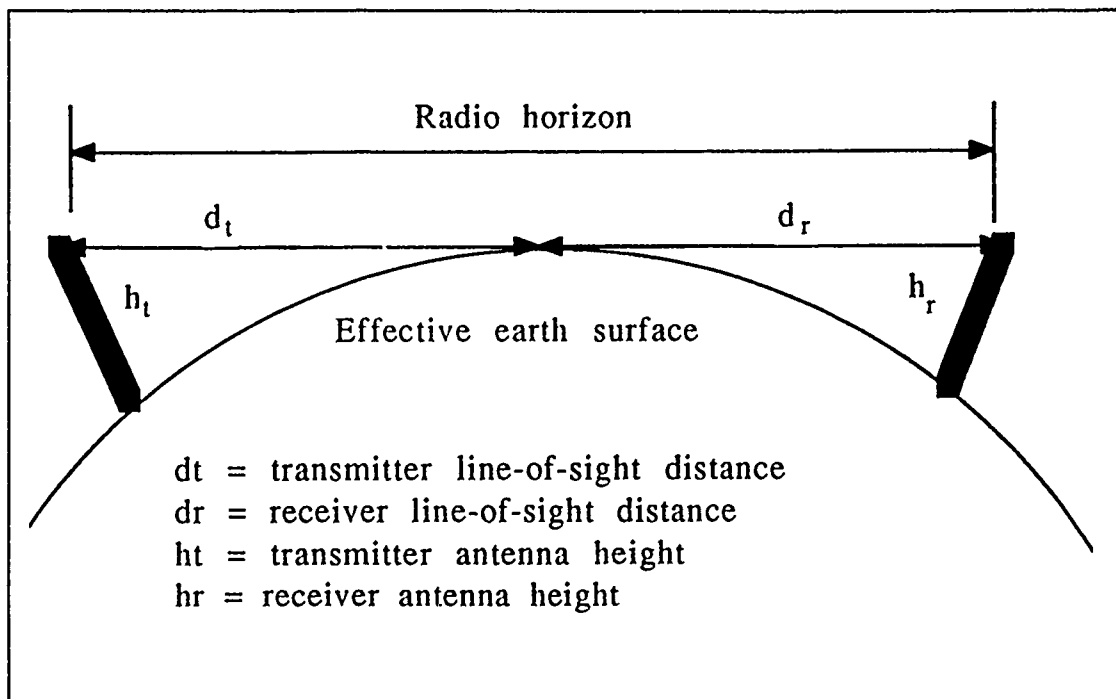
Figure 5. Graphical description of the radio horizon

frequency and the range) and the noise power, which is a function of the receiver acceptance bandwidth. The interception of the available SNR, expressed in decibels, is given by the vertical distance between the signal line and the noise line as indicated in Fig.5.

The minimum detectable signal, S_{min}, or the receiver sensitivity, is defined as the minimum SNR at the receiver input, multiplied by noise power of the receiver acceptance bandwidth. This relationship can be expressed as follows.

$$S_{min} = F_N kTB(SNR)_{min}$$

where $(SNR)_{min}$ is the minimum SNR.

7. Probability of False Alarm

In COMINT receivers, the probability of false alarm is calculated assuming the input to be noise only. Assuming the input noise to be Gaussian, one can show that the probability of false alarm is given by Ref. 12.

$$P_{fa} = \exp\left(-\frac{V_T^2}{2\psi_0} \right) \tag{2.1}$$
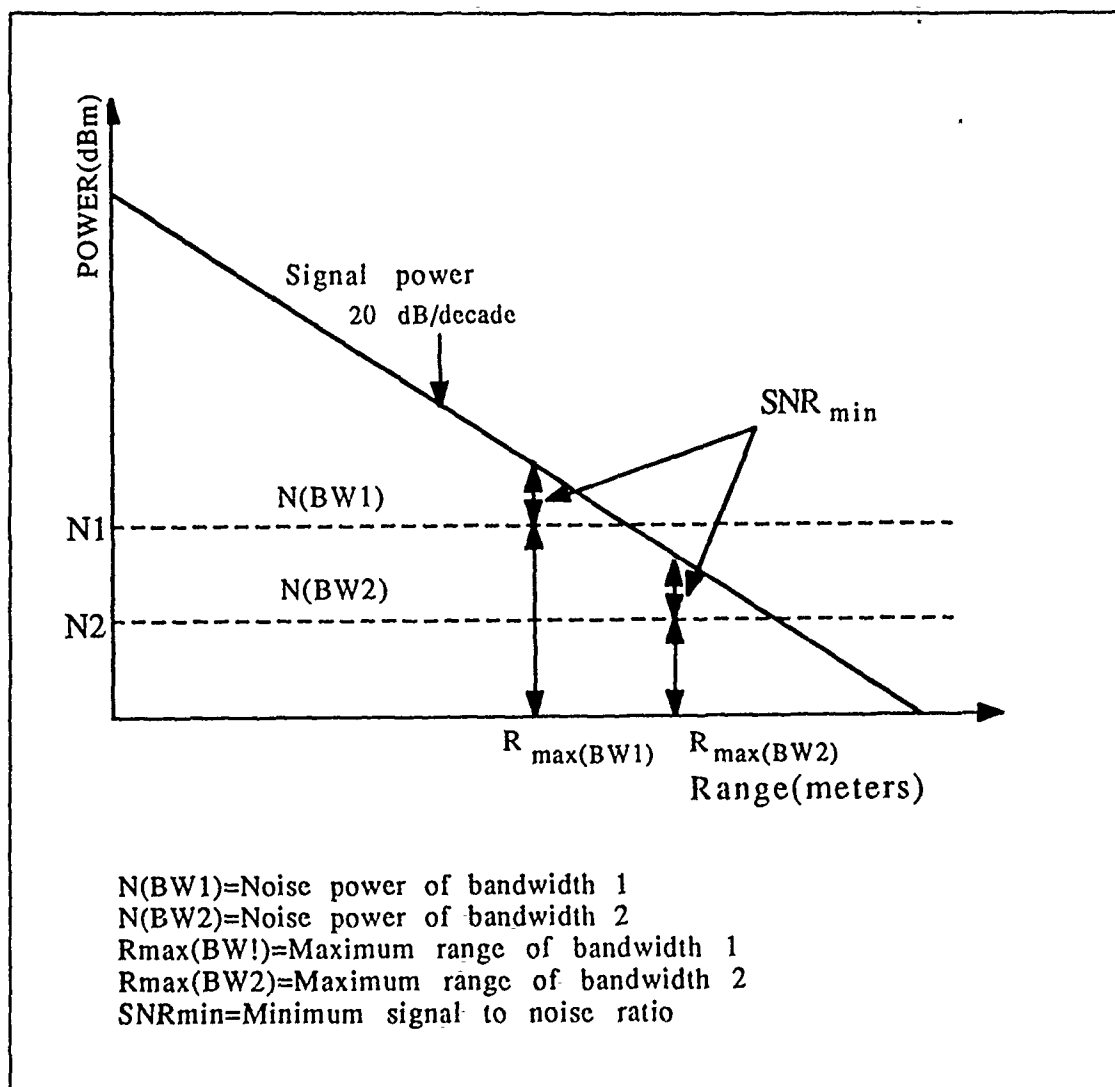
Figure 6. Graphical description of ESM line

where V_T is the preestablished threshold voltage, and ψ_0 is the variance, or mean square value of the noise voltage. The value of $\dfrac{V_T^2}{2\psi_0}$ is analogous to the signal to noise voltage ratio, using the threshold voltage V_T, to represent signal voltage.

The signal detection process in most intercept receivers is described in terms of threshold detection. Almost all detection decisions are based upon a comparison of the output of a receiver with some threshold level. If the envelope of the receiver output exceeds a preestablished threshold, a signal is said to be present. The threshold detector

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allows a choice between one of two hypotheses. One hypothesis is that the receiver output is due to the noise only; the other is that the output is due to signal-plus-noise.

Two types of error may be made in this decision process. One is to mistake noise for a signal when only noise is present while the other is to erroneously consider signal to be noise. The former is called a type I error while the latter is called a type II error.

This threshold detection is selected so as not to exceed a specified false alarm probability, that is, the probability of detection is maximized for a fixed probability of false alarm. This is equivalent to fixing the probability of type I errors which occur when noise exceeds the threshold creating a false alarm, and minimizing type II errors which occur when noise reduces signal below threshold for a missed detection. So it is similar to the Neyman-pearson test used in statistics for determining the validity of a specified statistical hypothesis.[Ref. 12] Therefore, this type of threshold detector is sometimes called a Neyman-Pearson detector.

Neyman-Pearson criterion provides the uniformly most powerful statistically based test for obtaining an indication of the case when a signal exceeds the threshold. Tests other than Neyman-Pearson lead to a higher probability of error for a given SNR [Ref. 12]. The Neyman-Pearson criterion is well suited to the intercept receiver work since it directly leads into the probability of detection and the probability of false alarm discussions.

The probability of false alarm is very important in radar ESM receivers since every false alarm is displayed as an intercept. Excessive false alarms generate unnecessary input data, degrading the ESM processors ability to sort and identify signals of interest. The effect of probability of false alarm on the overall performance of an ESM system was analyzed in Nicholson[Ref. 13] where he makes use of the Bayes theorem. Nicholson shows that the threat warning systems require the probability of false alarm much smaller than 1×10^{-4} in order to avoid excessive signal classification error.

In COMINT operation, the significance of the probability of false alarm is less than that in radar, since the communication intelligence data is not as time critical as radar ESM data. But one has to consider the effect of the probability of false alarm on the probability of detection, since the probability of detection is explained as a function of false alarm probability.

III. INTERCEPT PROBABILITY CONCEPT

A. DEFINITION OF PROBLEM

Intercept of a signal is one form of reconnaissance and possess many of the characteristics of reconnaissance systems in general. Reconnaissance is a collection of information on the facilities, capabilities and intentions of potential or actual hostile forces.[Ref. 14] The mission of reconnaissance is to measure the effectiveness of these facilities; to estimate their reliability; and to determine deployment and changes in the enemy's strategy and tactics.

COMINT differs significantly from ELINT. For one thing, the amount of information required for a successful COMINT operation is much greater than that required for a successful ELINT operation. Even though there is a significant difference between message reception in COMINT and ELINT, it is essential to the success of both operations to obtain knowledge of the possible disposition of a hostile presence as early as possible. To do this, an intercept system with a high intercept probability is required.

The probability that a given signal is detected and processed, or POI, is a function of both the signal and the receiving system. The ideal system should intercept any signal emitted within the maximum range based on free space attenuation factors, system sensitivity and terrain masking. We can imagine a system with high sensitivity, low probability of false alarm, wide RF bandwidth, 360 degree antenna coverage, large processing capacity, being reliable, economical and having the POI of unity. Obviously any receiver meeting all those requirements does not exist. The design of an intercept system has trade-offs between these various factors.

To analyze an intercept system, considering POI, we should realize that POI is largely a matter of definition based on particular purposes. The definition of POI given by Wiley[Ref. 15] is that "the joint probability of three independent probabilities such as; the probability that the receiver is tuned to the carrier frequency of the emitter, the probability that the antenna is pointed toward the emitter, and the probability that the emitter antenna is pointed toward the ESM station". This in itself is not a completely satisfying definition when one wishes to use it to evaluate the dynamic situation of signal environments and intercept systems competing against each other. Also, in a practical communication situation, the antenna for both the emitter and the receiver usually uses

an omni-directional antenna. In such a situation, the second and third cases would be eliminated.

Ortiz[Ref. 16] derived a mathematical model for the scenario of airborne radars and ESM receivers at a point in time and space. In this model, POI is defined as the product of probabilities of three independent events, which are; the probability of signal detection from the noise, the probability of coincidence between the emitter and the receiver, and the probability of identification of the emitter by the receiver processor or the operating system. This definition is quite reasonable and practical in the radar ESM.

In this thesis, we deal with COMINT, which concentrates on the reception of the communication messages of hostile forces. Here, the POI is defined as the product of two independent probabilities which are; the probability of detection of signal from the noise, and the probability of coincidence of various parameters between the emitter and the receiver. The definition of detection probability is adopted from Ortiz [Ref. 16], since the behavior of a signal in the atmosphere is quite similar in both radar and communications. The probability of coincidence in this thesis is defined as the product of two independent probabilities which are the probability of the transmitter-on and the probability of observation as a function of time, which is the probability that the intercept receiver is tuned to that frequency during the same time and both antennas look at each other at the same time. Therefore the definition of POI becomes:

$$P_I(t) = P_d \times P_c(t)$$

where

$P_I(t)$ = probability of intercept at time t

P_d = probability of detection of signal from the noise

$P_c(t)$ = probability of coincidence of frequency at time t

B. SIGNAL DETECTION FROM THE NOISE

Probability of detection (P_d) is a measure of the receiver's capability to detect a signal in the presence of noise. Signal detection in the presence of noise is equivalent to deciding whether the receiver output is due to noise alone or to signal-plus-noise. When detection is performed by automatic electronic tuning, it cannot be left to chance, but must be specified and built into the decision-making device by the system engineer. Here the signal detection process is described in terms of threshold detection, or in other words, Neyman-Pearson detection.

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If the form of a signal is known exactly, the probability of intercepting such a signal can be reduced to the probability of detecting it in the presence of noise. The probability of detection, in a threshold detector, may be expressed as a function of the probability of false alarm, which one is willing to tolerate, and the signal-to-noise ratio. If the detection threshold level is raised to decrease the probability of false alarm, the probability of detection will also decrease. The converse is also true. In other words, if we decrease the detection threshold level to increase the probability of the signal levels crossing the threshold, the probability of false alarm will also increase.

Skolnik[Ref. 12] develops a simple formula for the probability of detection which is a function of false alarm rate and the signal to noise ratio. This derivation is done by assuming that the Gaussian noise is passing through the receiver's narrow band IF filter. Using series approximation, the derived formula is expressed as follows: [Ref. 12: p.27]

$$P_d = \frac{1}{2}\left\{1 - erf\left(\frac{V_T - A}{\sqrt{2\psi_0}}\right)\right\}$$
$$+ \frac{\exp[-(V_T - A)^2/2\psi_0]}{2\sqrt{2\pi}\,(A/\sqrt{\psi_0})}\left[1 - \frac{V_T - A}{4A} + \frac{1 + (V_T - A)^2/\psi_0}{8A^2/\psi_0} - \cdots\right]$$

where the error function is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$$

This equation may be converted to power by replacing signal-to-rms-noise-voltage ratio with the following:

$$\frac{A}{\sqrt{\psi_0}} = \frac{signal\ amplitude}{rms\ noise\ voltage} = \frac{\sqrt{2}\ (rms\ signal\ voltage)}{rms\ noise\ voltage} = \left(2\,\frac{signal\ power}{noise\ power}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

We shall also replace $V_T^2/2\psi_0$ by $\ln(1/P_{fa})$ [from Equation (2.1)]. Using the above relationships, the probability of detection can be rewritten as follows: In order to express this equation in the form of the signal-to-noise ratio and the false alarm probability, we replace $V_T/\sqrt{2\psi_0}$ by $\sqrt{\ln(1/P_{fa})}$ and $A/\sqrt{2\psi_0}$ by $\sqrt{S/N}$, then

$$P_d = \frac{1}{2}\left\{1 - erf\left(\sqrt{\ln\left(\frac{1}{P_{fa}}\right)} - \sqrt{\frac{S}{N}}\right)\right\} + \frac{\exp\left\{-\left(\sqrt{\ln\left(\frac{1}{P_{fa}}\right)} - \sqrt{\frac{S}{N}}\right)^2\right\}}{4\sqrt{\pi}\sqrt{\frac{S}{N}}}$$

$$\times \left[0.75 - \frac{\sqrt{\ln\left(\frac{1}{P_{fa}}\right)}}{4\sqrt{\frac{S}{N}}} + \frac{1 + 2\left(\sqrt{\ln\left(\frac{1}{P_{fa}}\right)} - \sqrt{\frac{S}{N}}\right)^2}{16\left(\frac{S}{N}\right)}\right]$$

The derivation of this equation is done by Ortiz [Ref. 16] and we found his work reasonable and accurate.

This is the final form of the equation for the probability of detection in forms of signal-to-noise ratio and the probability of false alarm. In order to demonstrate that this formula is valuable, we introduce Tsui's equation [Ref. 10: pp.24-42] for the probability of detection, which deals with the effects of video bandwidth B_v. Since most superheterodyne receivers have approximately the same RF and video bandwidth, the ratio of the video bandwidth to the RF bandwidth $\frac{B_v}{B_r}$ for superheterodyne receiver is approximately unity. So the results of both equations should be reasonably close to each other in the case of superheterodyne receivers. Tsui's equation for the probability of detection is:

For $P_d < 0.5$,

$$P_d = \frac{B}{A}$$

For $P_d > 0.5$,

$$P_d = \frac{(A - C)}{A}$$

where

$$A = \frac{1}{\sqrt{2\pi}}\frac{K_3}{6K_2^{3/2}}(K_4 - 1)\exp\left(-\frac{K_4}{2}\right) + \left[\frac{1}{2} - \frac{1}{2}erf\left(-\sqrt{\frac{K_4}{2}}\right)\right]$$

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$$B = \frac{1}{\sqrt{2\pi}} \frac{K_3}{6K_2^{3/2}} (K_5^2 - 1) \exp\left(-\frac{K_5^2}{2}\right) + \left[\frac{1}{2} - \frac{1}{2} erf\left(\frac{K_5}{\sqrt{2}}\right)\right]$$

$$C = -\frac{1}{\sqrt{2\pi}} \frac{K_3}{6K_2^{3/2}} (K_5^2 - 1) \exp\left(-\frac{K_5^2}{2}\right) + \frac{1}{\sqrt{2\pi}} \frac{K_3}{6K_2^{3/2}} (K_4 - 1) \exp\left(-\frac{K_4}{2}\right)$$
$$+ \left[\frac{1}{2} erf\left(\frac{K_5}{\sqrt{2}}\right) - \frac{1}{2} erf\left(-\sqrt{\frac{K_4}{2}}\right)\right]$$

and

$$K_1 = 1 + \left(\frac{S}{N}\right)$$

$$K_2 = \frac{1}{(1 + \gamma^2/2)^{1/2}} \left[1 + \frac{S}{N}\left(\frac{1 + \gamma^2/2}{1 + \gamma^2/4}\right)\right]$$

$$K_3 = \frac{4}{2 + 3\gamma^2/4} \left[1 + 3\frac{S}{N}\left(\frac{2 + 3\gamma^2/4}{2 + \gamma^2/4}\right)^{1/2}\right]$$

$$K_4 = \frac{K_1^2}{K_2}$$

$$K_5 = \frac{(V_T - K_1)}{\sqrt{K_2}} \tag{3.1}$$

where $\gamma = \frac{B_r}{B_v}$ and here is approximately unity. In equation (3.1), assuming $\psi_0 = 1.0$, V_T can be determined from the given probability of false alarm, which is:

$$V_T = \sqrt{\ln\left(\frac{1}{P_{fa}}\right)}$$

Two other statistical criteria, usually discussed when considering detection of targets in noise, are the likelihood ratio and the inverse probability; but these types of receivers are seldom implemented in practice.[Ref. 12] In some cases, the receiver which computes the likelihood ratio is equivalent to one which computes the cross-correlation function, or one with a matched-filter characteristic. The inverse probability receiver requires that the probability of a target being present in a particular range cell must be known a

priori. In practical situations, this is rarely possible. Thus, this type of receiver is difficult to implement.

C. COINCIDENCE CONCEPT

The previous section introduced the concept of probability of signal detection when greater degree of uncertainty of noise is involved. If the signal has sufficient strength to cross the threshold and coincidence does not occur between the signal parameters and the receiver parameters, the signal will not be intercepted.

The main point of concern here is determining the probability that an intercept system is available, actively listening in the proper frequency band, in the right direction and at the same time that the signal is received. Here it is assumed that there is adequate signal energy available to the intercept receiver input such that the probability of detection is nearly unity and the probability of false alarm is very small.

Considering the typical situation in which an emitter is radiating, a narrow band superhet receiver is tuned to frequency across a band containing the signal of interest, and a narrow beam parabolic antenna concurrently looks for the signal of interest, in space.

We are concerned with the joint occurence or coincidence of those independent events. Coincidence determines whether or not the signal will be intercepted. So the intercept problem can be reduced to finding the probability of coincidence of those events.

It is convenient to represent these events in frequency and angle as *window functions*. Fig.7 shows a time-frequency diagram for a receiver scanning the frequency band from f_{min} to f_{max} with a linear sawtooth sweep. This figure can be used to calculate the probability of coincidence, in both time and frequency, between a periodic signal and the tunable receiver's frequency acceptance band.

Coincidence calculations have been formed on the basis of the intercept probability estimates done by Boyd [Ref. 14], wiley [Ref 15] and Schlesinger [Ref. 17]. One of more recent work in this area is by Wiley[Ref. 15]. His work is based on the periodic nature of pulse radar signal, rotating antenna and scanning receiver, which can be represented as periodic window functions.

After necessary deleberation, we found that Wiley's equation for the probability of coincidence is not applicable for the communication scenario, since most of the time, the period of receiver scan and the antenna scan is much smaller than the signal duration. In this thesis, after some modification of the equation for probability of interception

developed by Schlesinger[Ref. 17], we choose to introduce a different approach for the coincidence problem.

Here, the consideration for probability of coincidence is the probability that the desired emitter is operating during the period when observation is possible. So the probability of coincidence can be represented as the product of probability of the transmitter-on and the probability of observation as a function of time. In order to observe the desired signal, two scanning characteristics should coincide each other. We can represent this coincidence factor of two scanning characteristics as β which is:

$$\beta = K \frac{\tau_s \tau_a}{T_s T_a}$$

where

β = coincidence factor

K = correction factor

τ_s = receiver dwell time in seconds

τ_a = antenna illumination time in seconds

T_s = receiver scan period in seconds

T_a = antenna scan time in seconds

τ_s/T_s = duty factor of the receiver ($= B_r/D_s$)

τ_a/T_a = duty factor of the antenna ($= \theta/360°$)

B_r = receiver acceptance bandwidth in hertz

D_s = frequency coverage of interest in hertz

θ = receiver antenna beamwidth in degree

Since the emitter does not operate in the desired periodic manner, for simplicity, we will assume that the mean signal duration is τ_{on} and the mean time between signals is τ_{off}. The probability of the desired signal being on at any given time, can now be given as:

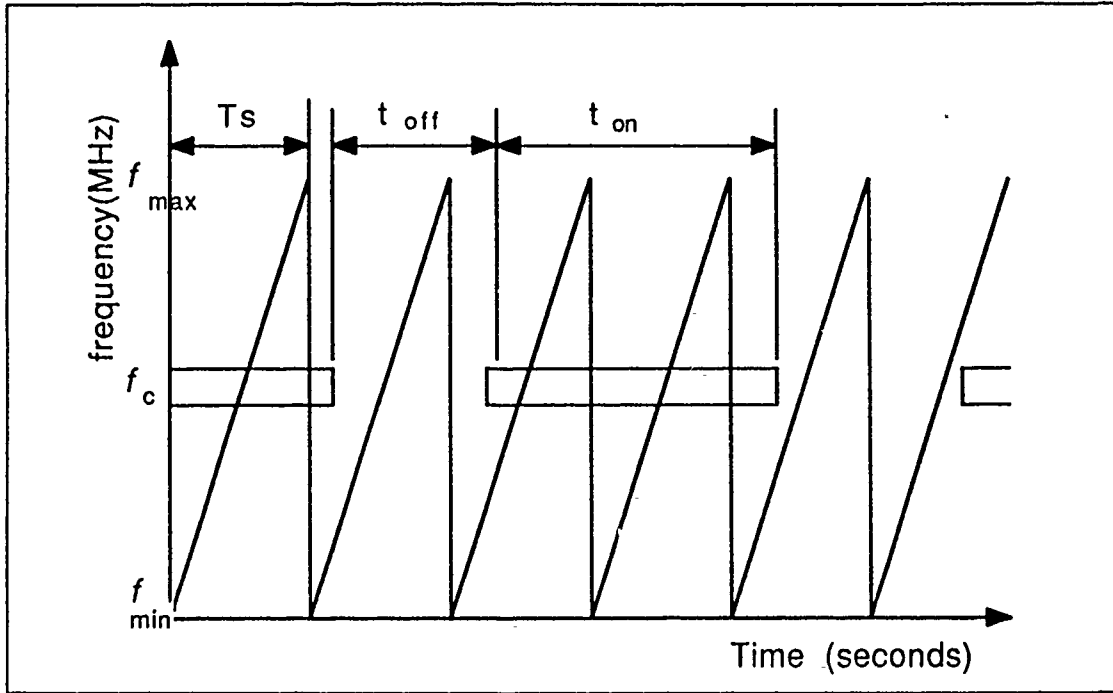$$P_{on} = \frac{\tau_{on}}{\tau_{on} + \tau_{off}} \tag{3.2}$$

where

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Figure 7. Time-frequency diagram for scanning receiver

P_{on} = probability of transmitter-on

τ_{on} = mean signal duration in seconds

τ_{off} = mean time between signal exposure in seconds

Similarly, the probability of observing the signal by continously looking during the time t is a function of

$$\int_0^t \beta dt \qquad (3.3)$$

where, β is defined as the coincidence factor. Assuming β is constant, Equation (4.3) is explained as follows.

If there is no observation during the interval $t + dt$, then observation must fail during both of the intervals, t and dt, Let the probability of not observing during $t + dt$ be $\lambda(t + dt)$; the probability of not observing during the period t is $\lambda(t)$; and for the period dt is $1 - \beta dt$. These relationships, assuming independence, can be expressed as

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$$\lambda(t + dt) = \lambda(t)(1 - \beta dt)$$

which yields the differentail equation

$$\frac{d\lambda(t)}{dt} = -\beta\lambda(t)$$

Rearranging and integrating

$$\int \frac{d\lambda(t)}{\lambda(t)} = -\int \beta dt$$

gives

$$\ln \lambda(t) = -\beta t$$

or

$$\lambda(t) = e^{-\beta t}$$

Therefore, the probability of observing the signal during the time period t can be written as:

$$P_{ob}(t) = 1 - e^{-\beta t} \tag{3.4}$$

Considering the results of Equations (3.2) and (3.4), the probability of coincidence during the looking period time t, under this condition, is:

$$P_c(t) = P_{on} \times P_{ob}(t)$$

or

$$P_c(t) = P_{on}\left(1 - e^{-\beta t}\right)$$

For $t = 0$, $P_c(t) = 0$, which indicates that if no time is spent looking, probability of coincidence is zero. Also if the on-time of the transmitter, τ_a, equals to zero, $P_c(t)$ equals to zero, which indicates that if the emitter is off, it obviously cannot be coincided.

This gives the probability of coincidence of emitter and receiver, operating as difined above, and assuming that the signal of interest has sufficient strength to cross the

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threshold. In communication scenario, if the sweep periods of both receiver and antenna are fast enough than the signal duration, we may guarantee the 100 % signal interception which cross the threshold. That means, if we have fast scanning receiver and fast rotating directional antenna or a very sensitive omni-directional antenna, the probability of coincidence should be a certainty. Otherwise, this equation derived here will give an approximate answer which can evaluate the system performance from the parameters of a given scenario.

D. INTERCEPT PROBABILITY FOR SINGLE INTERCEPT SYSTEM

Based on the results of various calculation for the probability of detection and the probability of coincidence, we can obtain the final form of the probability of intercept for a single intercept system. In practice, operation of single intercept system is not realistic. Usually, there are two or more intercept system being operated. In order to understand the basic concept of the probability theory, we calculate the fundamental intercept probability:

$$P_I(t) = P_d \times P_c(t)$$

or

$$P_I(t) = P_d \times P_{on} \times P_{ob}(t)$$

The probability of detection, P_d, is evaluated based on the signal-to-noise ratio and the probability of false alarm. The probability of coincidence, $P_c(t)$, is a function of the geometry of the intercept system and the hardware characteristics of the emitter and intercept system.

A worksheet is developed in Appendix D, to illustrate the evaluation of this equation, and the analysis of the effects of various parameters is done in the next chapter.

E. INTERCEPT PROBABILITY FOR MULTIPLE INTERCEPT SYSTEMS

We assume that the signal intercept system behaves probabilistically in the sense that, when a signal of interest appears in the environment for some period of time the probability that it will be intercepted is not one. There is no absolute guarantee that it will be intercepted, therefore the system will not necessarily respond in a completely deterministic way.

The assumptions for the discussion of the problem are as follows:

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1. There are two or more intercept systems excited by a common emitter source.

2. The receiving systems are colocated and tied to a common antenna, in which case they share a common channel.

3. The intercept probabilities of the intercept systems are different and the systems behave independently in a statistical manner.

Based on these assumptions, the visualization of the environment is shown in Fig.8. The following discussion is based on Ref. 18.

If a signal of interest is transmitted, it will be intercepted by the receiving systems in some combination or not at all. We wish to determine the probability of a specified number of simultaneous intercepts. Let us define $P(N)$ as the probability of N simultaneous intercepts where $N \in \{0,1,2,\ldots,n\}$ and where n is the total number of the intercept systems. Also define P_{Ij} as the intercept probability of the jth intercept system. For the moment let us assume that the P_{Ij} are known; then the calculation of $P(N)$ is given as follows:

1. Calculate $x_j = P_{Ij}/(1 - P_{Ij})$

2. Expand $\prod\limits_{j=1}^{n}(x - x_j)$ to obtain the polynomial

$$\prod\limits_{j=1}^{n}(x - x_j) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n$$

3. Calculate $P(N) = |a_N| / \sum\limits_{k=0}^{n} |a_k|$

The derivation of this algorithm is given in the Appendix B.

The mean and variance of this probability law can be calculated very simply. Let us define the random variables, $Y_j = 1$, when the intercept system, j, intercepts the signal, otherwise $Y_j = 0$. Then, each time a signal of interest is transmitted, the number of simultaneous intercepts is:

$$N = \sum\limits_{j=1}^{n} Y_j$$

The mean, or expected value of the number of simultaneous intercepts is then:
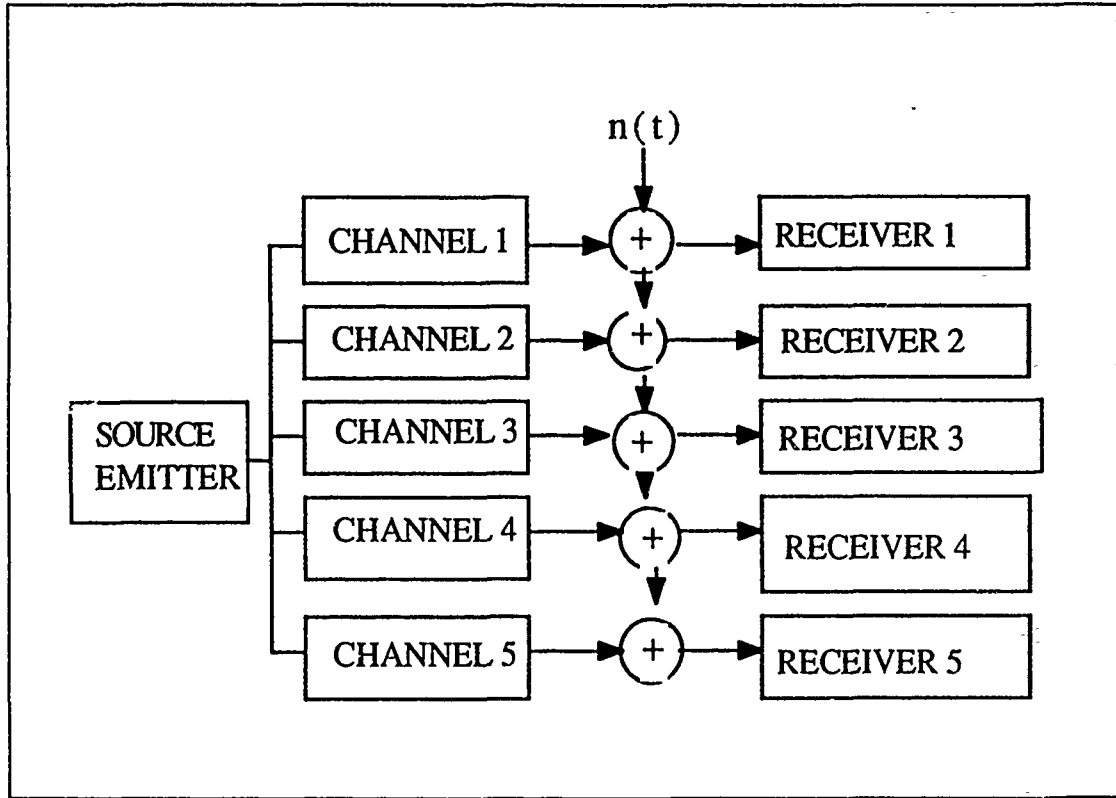
$$E[N] = \sum\limits_{j=1}^{n} P_{Ij}$$

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Figure 8. Graphical description of intercept systems for common antenna

Also the variance of this probability law is given by:

$$Var[n] = E[(N - \overline{N})^2] = \sum_{j=1}^{n} P_{Ij}(1 - P_{Ij})$$

where $\overline{N} = E[N]$.

If we consider a special case, that all the intercept systems have same probability of intercept, $P_{Ij} = P_I \forall j$, then this probability law reduces to the binomial case. [Ref. 18]

$$P(N) = \binom{n}{N} P_I^N (1 - P_I)^{n-N}$$

where $N \in \{0,1,2, \dots , n\}$ which has mean $E[N] = nP_I$ and variance $Var[n] = nP_I(1 - P_I)$.

Using this concept, we can calculate the probability that the signal of interest is not intercepted and the probability that more than one intercept occurs among the number

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of intercept systems. Since we know the formula for $P(N)$, the probability of not intercepting the signal of interest, $P(0)$, is:

$$P(0) = | a_0 | / \sum_{k=0}^{n} | a_k |$$

or, if the intercept systems have the same intercept probability, then

$$P(0) = (1 - P_I)^n$$

Also, the probability that intercept occurs in any one of the systems is

$$P_I(n) = P(N \geq 1) = 1 - P(0)$$
$$= 1 - (1 - P_I)^n$$

This result will be demonstrated graphically in the next chapter using MATHCAD.

F. TIME DEPENDENT PROBABILITIES

When one attempts to calculate the probability of intercept as a function of the duration of the searching time, one encounters various problems related to the periodic nature of the events under consideration.

Since the definition of probability of intercept is

$$P_I(t) = P_d \times P_{on} \times P_{ob}(t)$$

where $P_{ob}(t)$ equals $(1 - e^{-\beta t})$, we can rearrange this equation as:

$$e^{-\beta t} = 1 - P_{on} \frac{P_I(t)}{P_d}$$

where β is $K \frac{\tau_s \tau_a}{T_s T_a}$. Solving this equation for the time

$$t_{req} = -\frac{1}{K} \frac{T_s T_a}{\tau_s \tau_a} \left(1 - P_{on} \frac{P_I(t)}{P_d} \right)$$

Since the duty factors of the transmitter, the scanning superheterodyne receiver and the rotating antenna are fixed, we can reduce the time required to a certain intercept probability by increasing the probability of detection, which is a function of signal-to-noise ratio. That means, the key factor to increase the probability of intercept in the communication scenario is the signal-to-noise ratio.

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IV. ANALYSIS

A. GENERAL

The analysis of the equations developed in this thesis is accomplished using MATHCAD2.0, which is a high-level programming language equation solving software package. We find that this software is very convenient to use in solving the complex equations generated by this thesis without excessive programming effort.

One of the big advantages of MATHCAD is in its ability to solve and display complex equations and to write text and to make on-screen plots quickly and easily. Also this software supports more than 70 built-in functions, including various mathematical and statistical functions. Particularly useful to the analysis done in this thesis is the capability to evaluate the error function, $erf(x)$, which is used extensively in calculating the probability of detection. Another strength is the iteration capability. This capability made it possible to compare the various parameters in POI(t), by stepping through the parameter variations.

The structure of the analysis is as follows. All initial parameter settings are given in the SETUP file. The appication files do the following steps.

1. call SETUP file

2. assign parameter variables

3. calculate the POI as a function of the given parameters

4. plot the POI versus the given parameters

The detailed description of this flow diagram is shown in Figure 9.

There are following files in Appedices C through G.

- Appendix C - SETUP
  The user inputs all emitter and interceptor parameters in this file.

- Appendix D - POIT
  This file calculates the POI as a function of time under given setup condition.

- Appendix E(1) - POID
  This file calculates the POI to determine the multipath effect as a function of the distance between the emitter and the interceptor.

- Appendix E(2) - POIF
  This file calculates the POI to determine the m... ath effect as a function of the frequencies of the signal.
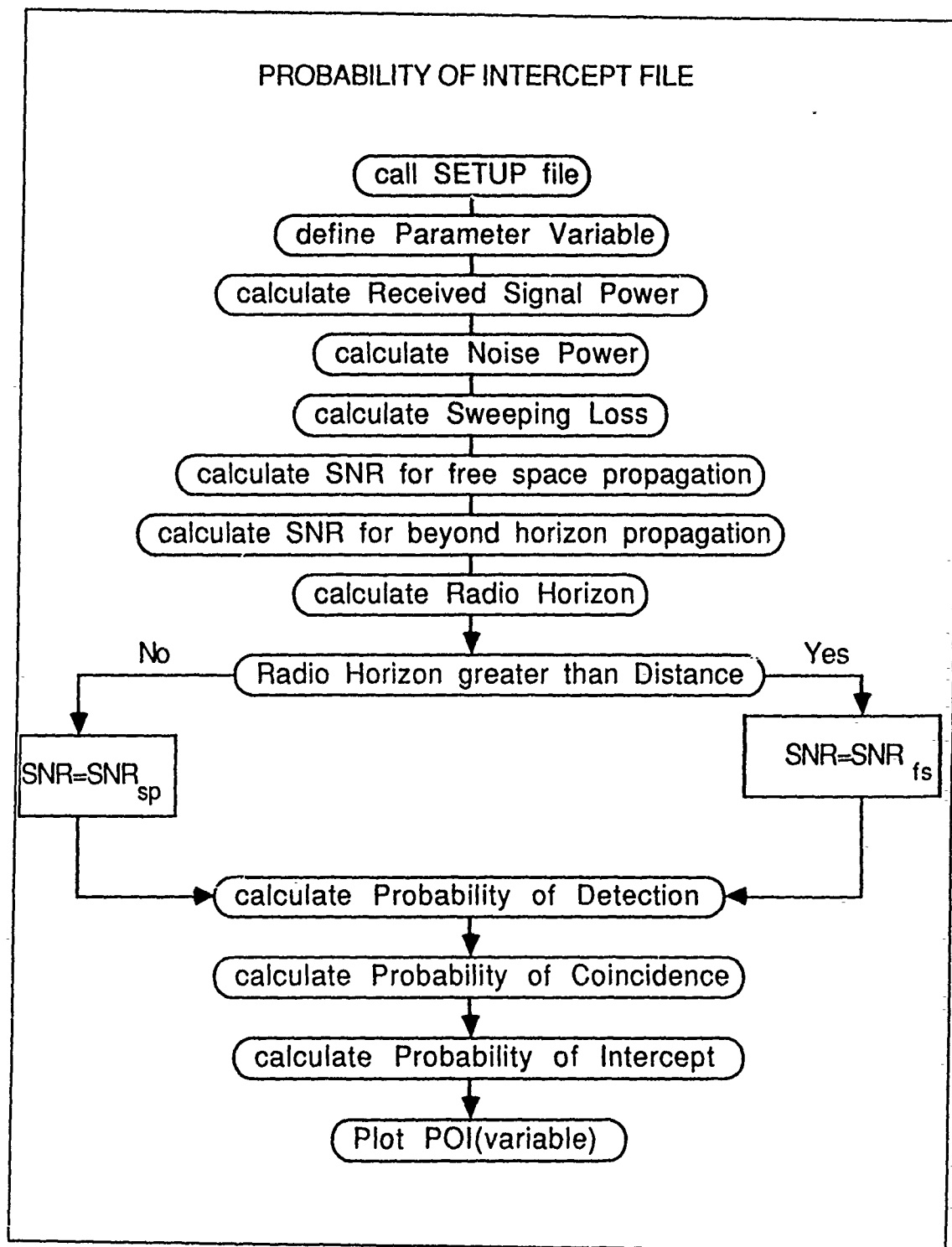
- Appendix E(3) - POIA

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Figure 9.   Flow diagram of the analysis file

This file calculates the POI to determine the multipath effect as a function of the receiving antenna height of the interceptor.

- Appendix F(1) - POIB
  This file calculates the POI to determine the receiver acceptance bandwidth effect as a function of the bandwidth of the interceptor.

- Appendix F(2) - POIV
  This file calculates the POI for a superhet receiver and a general type receiver to determine the video bandwidth effect.

- Appendix G - POIM
  This file calculates the POI for multiple system operation as a function of the number of interceptors.

B. PROBABILITY OF INTERCEPT AS A FUNCTION OF TIME

POIT can be easily used to predict the effects of varying Emitter/Interceptor parameters. Fig.10 shows the plot of POI(t) for a wide band, sweeping superheterodyne receiver versus the tactical communication emitters. The numerical values of the parameter settings are as follows:

- Radiated power from the emitter = 1 Watt

- Power gain of the emitter isotropic antenna = 1 (no unit)

- Center frequency of the signal = 50 MHz

- Distance between the emitter and the interceptor = 20 km

- Emitter antenna elevation above ground = 150 m

- Interceptor antenna elevation above ground = 150 m

- Receiver acceptance bandwidth = 10 MHz

- Receiver noise figure = 10 dB

- Total frequency coverage = 0 to 500 MHz

- Probability of false alarm = 10^{-4}

- Average operation time of the emitter = 5 seconds

- Average off-time of the emitter = 100 seconds

- Receiver dwell time = .02 seconds

- Receiver scan time = 1 second

- Antenna illumination time = .1 seconds

- Antenna scan time = 1 second

Some of these parameters affect the SNR only, and others affect the SNR and the probability of coincidence. There are many combinations of the parameter values. This section analyzes the POI versus several parameter values as a function of time.

Under the condition that the coincidence factor is fixed and the transmitted power increases from 10 miliwatts to 1000 milliwatts, we generate the POI plot as a function of time as shown in Fig.10. In this figure, we observe that the POI increases more quickly over time as the transmitter power increases. Fig.11 continues the conditions of Fig.10 with the coincidence being varied instead of the transmitted power which is now fixed at 1000 milliwatts. We observe that by increasing the coincidence factor, we can reduce the time required to obtain a certain POI and each curve converges to the same value given sufficient time. In this intercept time model, the key factor to improve the time basis POI is the coincidence factor.
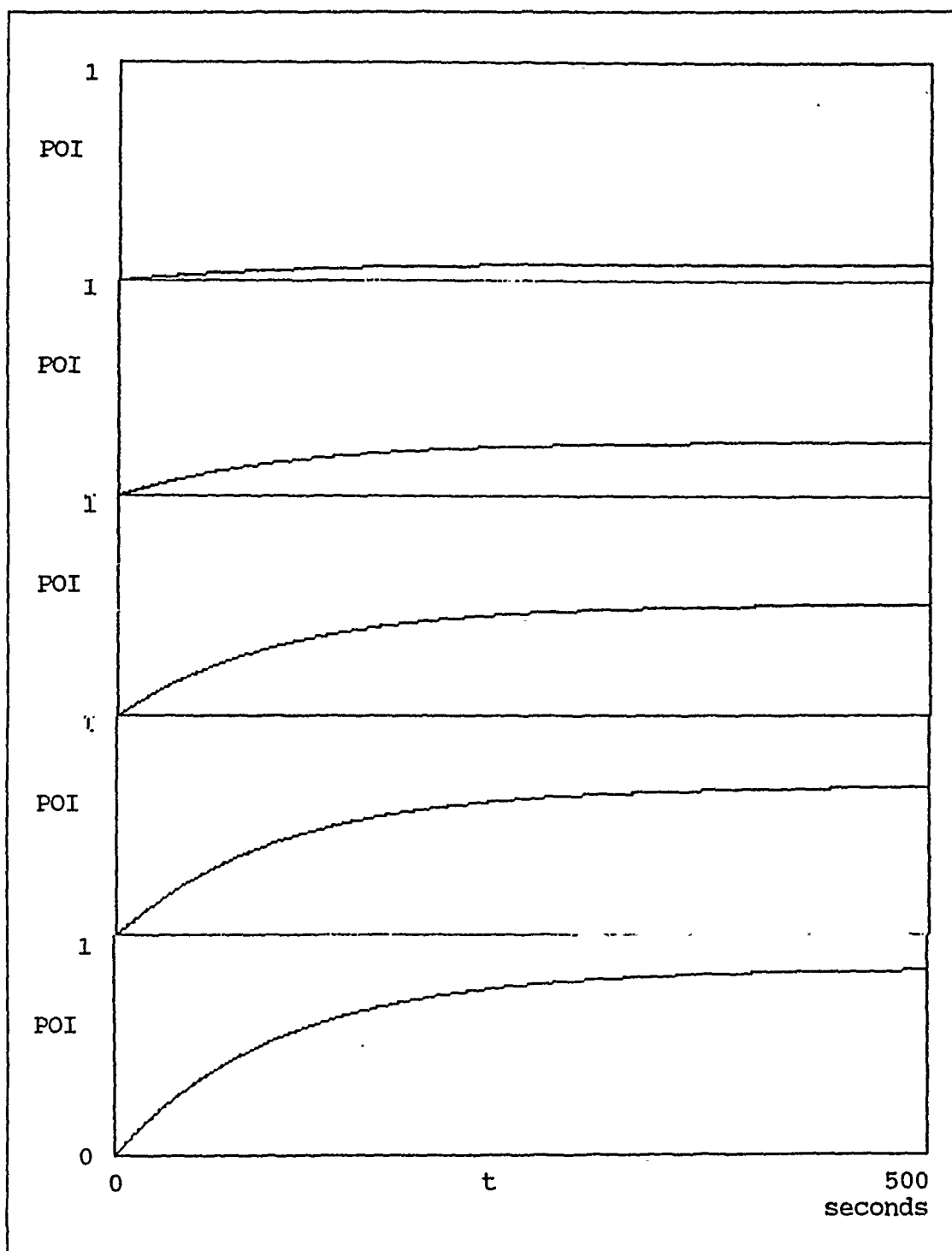
Figure 10.   POI^T plot(1): Intercept time with signal power variation: $\beta = 0.01$,
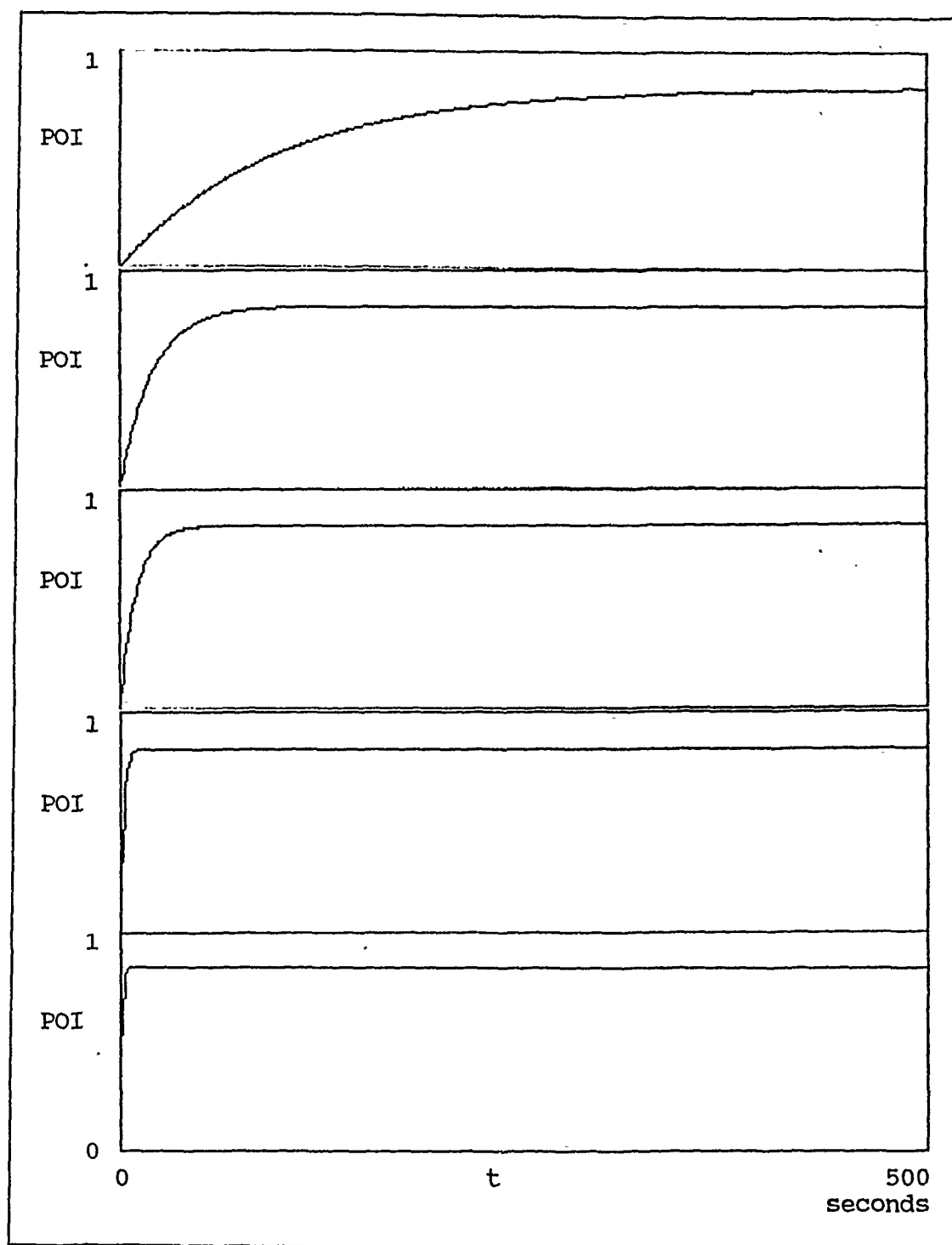$P_t = $ 10, 50, 100, 500, 1000 milliwatts

Figure 11. POIT plot(2): Intercept time with coincidence factor variation: $P_t = 1$ watt, $\beta = 0.01, 0.05, 0.1, 0.5, 1$

C. MULTIPATH EFFECT

The general features of the interference phenomena associated with antennas, separation distance and the frequency can be determined by studying the effects associated with these parameters. As shown in Fig.3, the direct ray and indirect ray reach the receiving antenna. When the two path lengths differ by an appropriate amount, there will be either constructive or destructive interference at the receiving antenna.

1. Probability of intercept as a function of the separation distance

POID can be used to measure the effect of the multipath phenomena, associated with the separation distance, in order to optimize the POI. By varying the separation distance between the emitter and the interceptor, we can find the locations conducive to constructive interference. Also, since the separation distance is one of the factors affecting path loss, we observe, as expected, that the POI degrades as the distance increases. The POID plot (POI as a function of the separation distance) is shown in Fig.12.
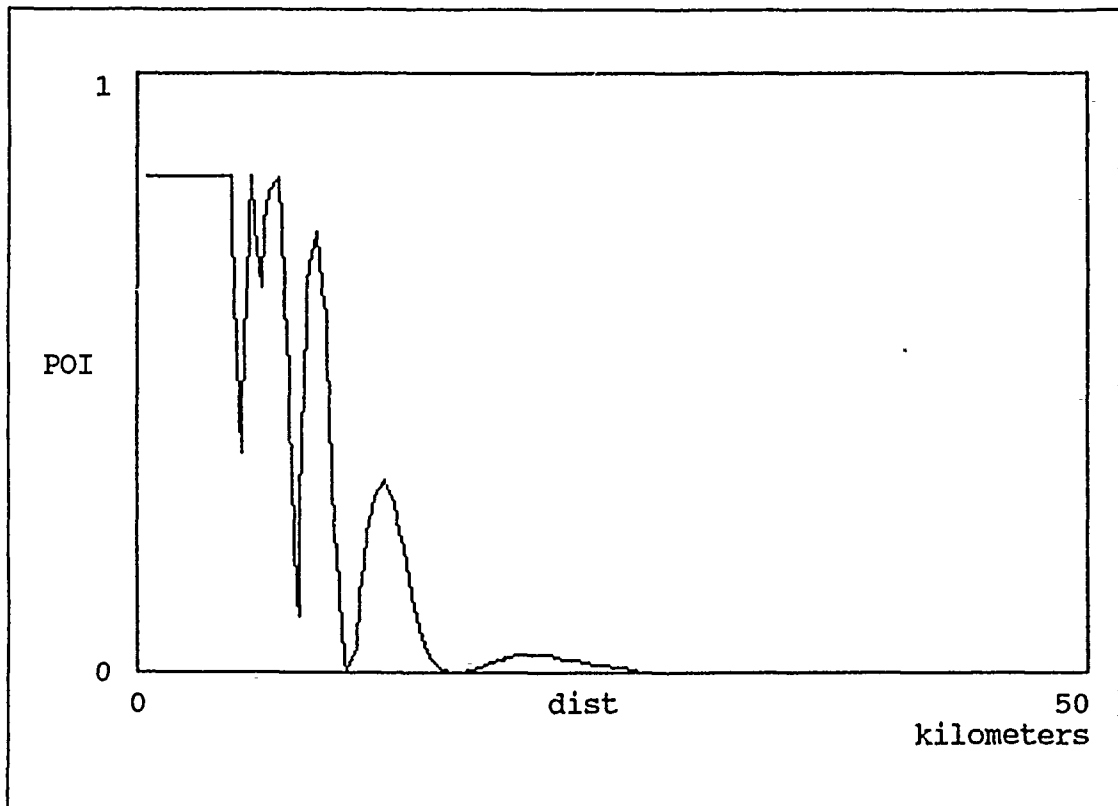

Figure 12.   The POID plot: The effect on multipath of varying the distance

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2. Probability of intercept as a function of the antenna elevation

POIA can be used to measure the effect of the multipath phenomena associated with the receiving antenna elevation. Under the conditions of fixed separation distance and frequency, a computer run was made to observe the variation of the POI as a function of antenna elevation variation. Once the identification of frequency and emitter location is determined, then one can optimize the POI by choosing the appropriate antenna elevation. The plot of POIA (POI as a function of an antenna elevation) is shown in Fig.13.
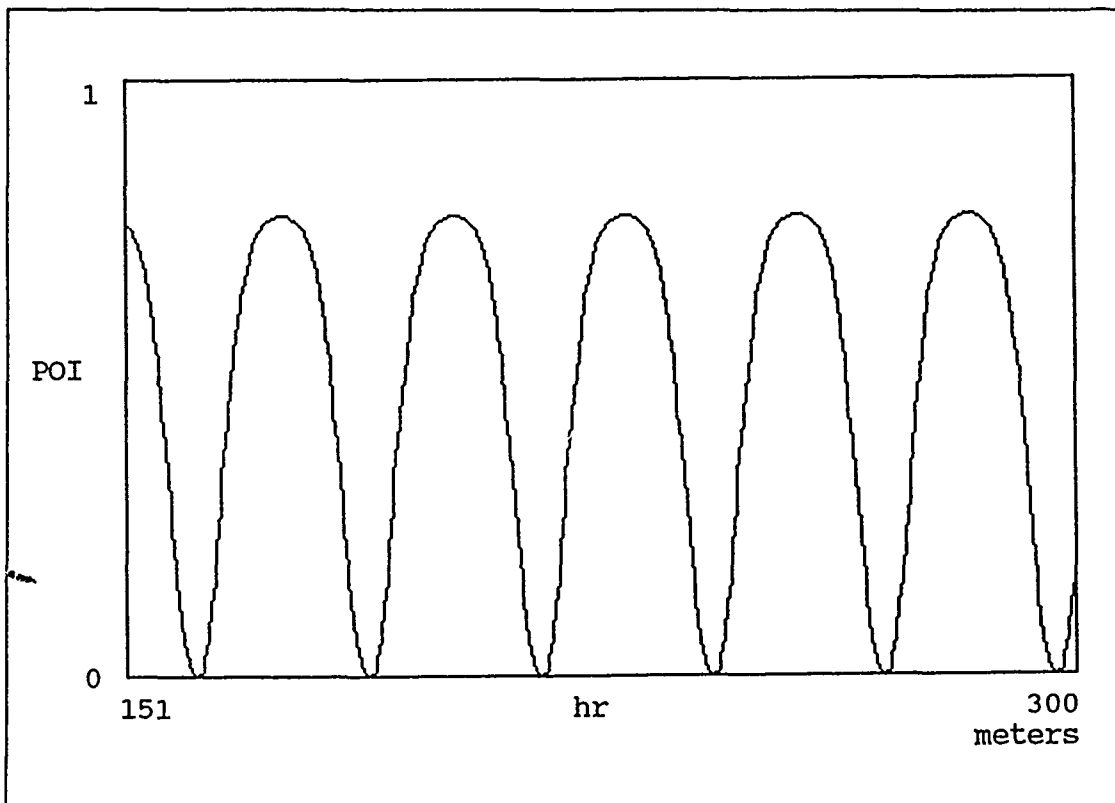

Figure 13. The POIA plot: The effect of the multipath by varying the antenna elevation.

3. Probability of intercept as a function of frequency

The file POIF can be used to predict the effect of the multipath phenomena associated with the frequency of the signal. The frequency transmitted by the hostile emitter is not controllable by the intercept site. However, we can optimize the POI by choosing the appropriate antenna elevation and the interceptor location according to the frequency. The frequency is also a factor in the path loss, since the path loss is

proportional to the square of the frequency, i.e., the higher the frequency the higher the path loss. However, the frequency also affects parabolic antenna gain factor. It is desirable to make tradeoffs between the gain and the loss at given antenna elevation. Fig.14 shows the POIF plot associated with the frequency variation. Under the conditions of given initial parameters, 50 MHz of the transmitted frequency provides less than the maximum value of the POI. Then we need to reset the elevation of the interceptor antenna to obtain the maximum POI.
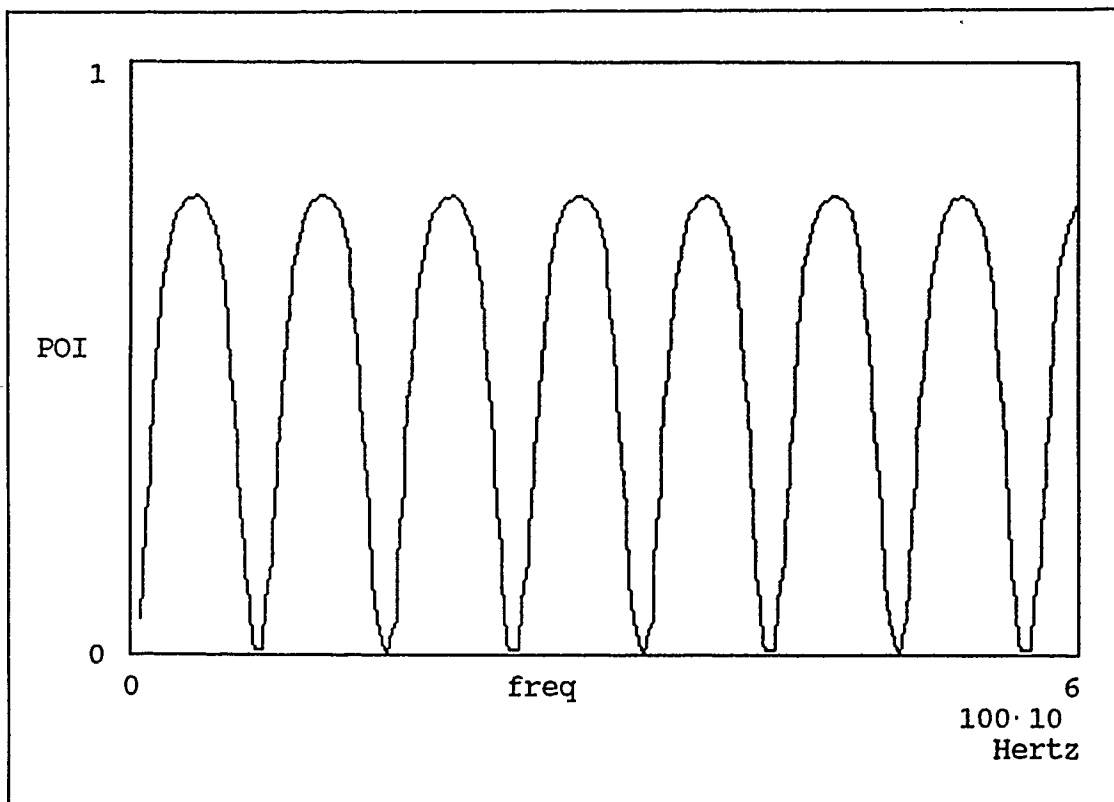

Figure 14.   The POIF plot: The probability of intercept as a function of the frequency

D. BANDWIDTH EFFECTS

The study of the intercept receiver characteristics reveals performance differences related to bandwidth, as defined earlier. The total frequency coverage, called D_t, describes the breadth of the total RF range over which the receiver can be operated. It defines the maximum bandwidth that can be assigned to a monitoring receiver.

The next consideration is the receiver acceptance bandwidth which may or may not coincide in numerical value with the total frequency coverage. It is the bandwidth over

which the receiver is instantaneously sensitive. In a wide open receiver, the receiver acceptance bandwidth corresponds to the total frequency coverage, since at any instant the receiver is equally responsive to signals anywhere in the total frequency range. In other receivers, the receiver acceptance bandwidth is less than the total frequency bandwidth. In a superheterodyne receiver, for example, the receiver acceptance bandwidth equals the IF bandwidth. It is also called the predetection bandwidth and is of prime importance in controlling the intercept probability of the receiver. The predetection bandwidth bears a direct relationship to the common receiver characteristics of selectivity and resolution. The ability to select one signal from a group of signals on a frequency difference basis, or to resolve two signals adjacent in frequency, is set in an intercept receiver by the value of the receiver acceptance bandwidth.

The video or postdetection bandwidth usually represents a design compromise influenced by requirements peculiar to intercept receivers. The selection must be consistent with the most severe requirement imposed by the need to reproduce to some degree the modulation waveshape of any baseband signals or class of signals anticipated for reception. If, for example, the video bandwidth is two small, a narrow pulse will not reach full amplitude. However, if the principal objective is only signal detection, a considerable reduction in video bandwidth is allowable for only a small loss in weak signal detectability, since there is a concomittant reduction of noise power bandwidth.

1. Probability of Intercept as a function of a receiver acceptance bandwidth

The total frequency coverage bandwidth we considered is 500 MHz . If we vary the receiver acceptance bandwidth, the noise power, receiver sweeping loss and the duty factor of the receiver vary. Fig.15, the POIB plot, shows the POI as a function of the receiver acceptance bandwidth.

Once the receiver acceptance bandwidth increases above 1 MHz, the probability of detection decreases , since the probabiiity of detection is a function of the SNR, and the receiver noise bandwidth is directly related to the acceptance bandwidth. However, the probability of coincidence increases with bandwidth, since the probability of coincidence is an inverse function of the scanning factor, and the scanning factor deceases when the receiver acceptance bandwidth increases. In this model, we can optimize the receiver acceptance bandwidth by choosing the value which results in the highest POI.
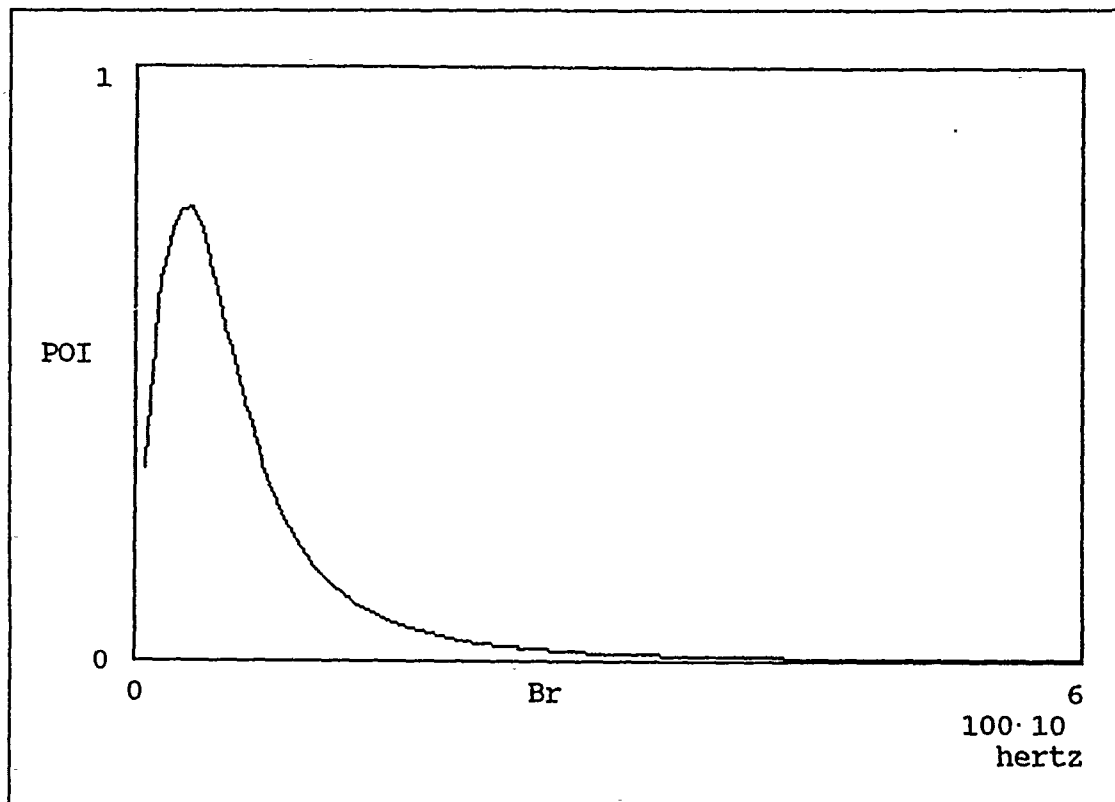
Figure 15. The POIB plot: The probability of intercept as a function of the receiver acceptance bandwidth

2. Video bandwidth effects

The file POIG can be used to predict the video bandwidth effect associated with the POI. Most superheterodyne receivers have approximately the same receiver acceptance bandwidth and video (baseband) bandwidth. Actually, for the study of the superheterodyne receiver, we could ignore the video bandwidth effect with negligible loss in accuracy.

For other receivers, a plot of POIG shows that the effect of the video bandwidth (varying the ratio of the receiver acceptance bandwidth to the video bandwidth, y). Fig.16 shows the POI as a function of the video bandwidth using equation 4.2.

In Fig.16 and 17, POIG actually decreases as y increases because with SNR less than 3 dB, the noise actually grows faster than the signal with increasing y .
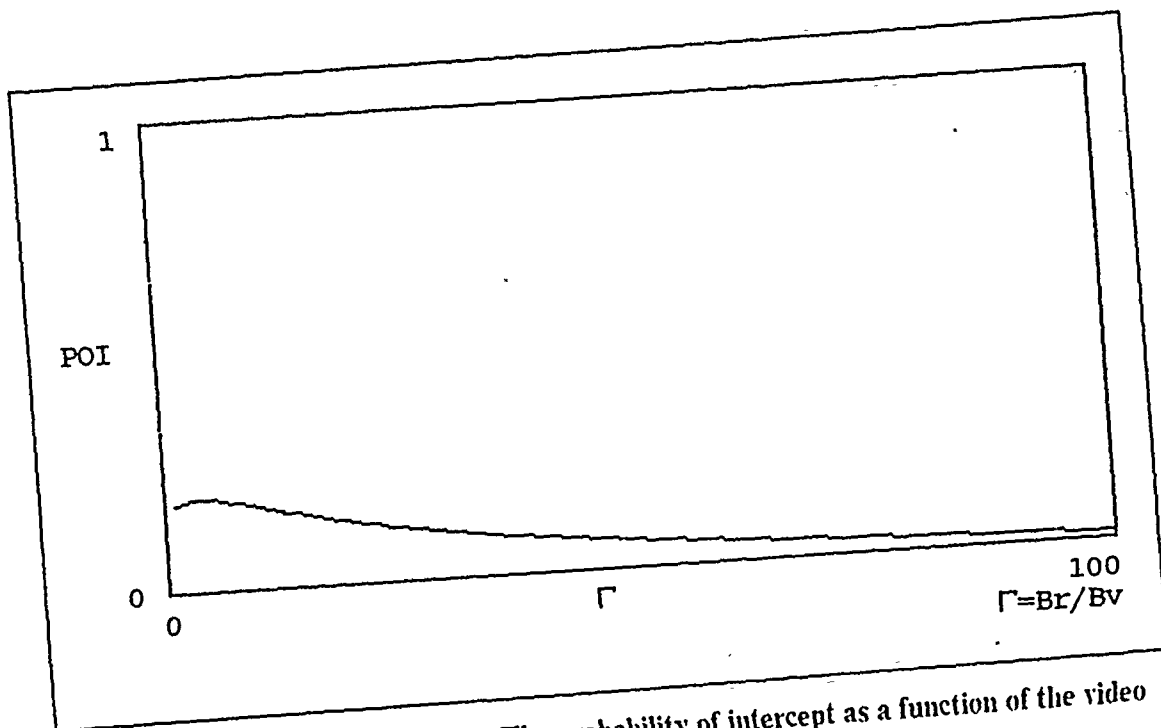
Figure 16. The POIG plot(1) - The probability of intercept as a function of the video bandwidth: $P_t = 10$ milliwatts. $(SNR \approx 2dB)$
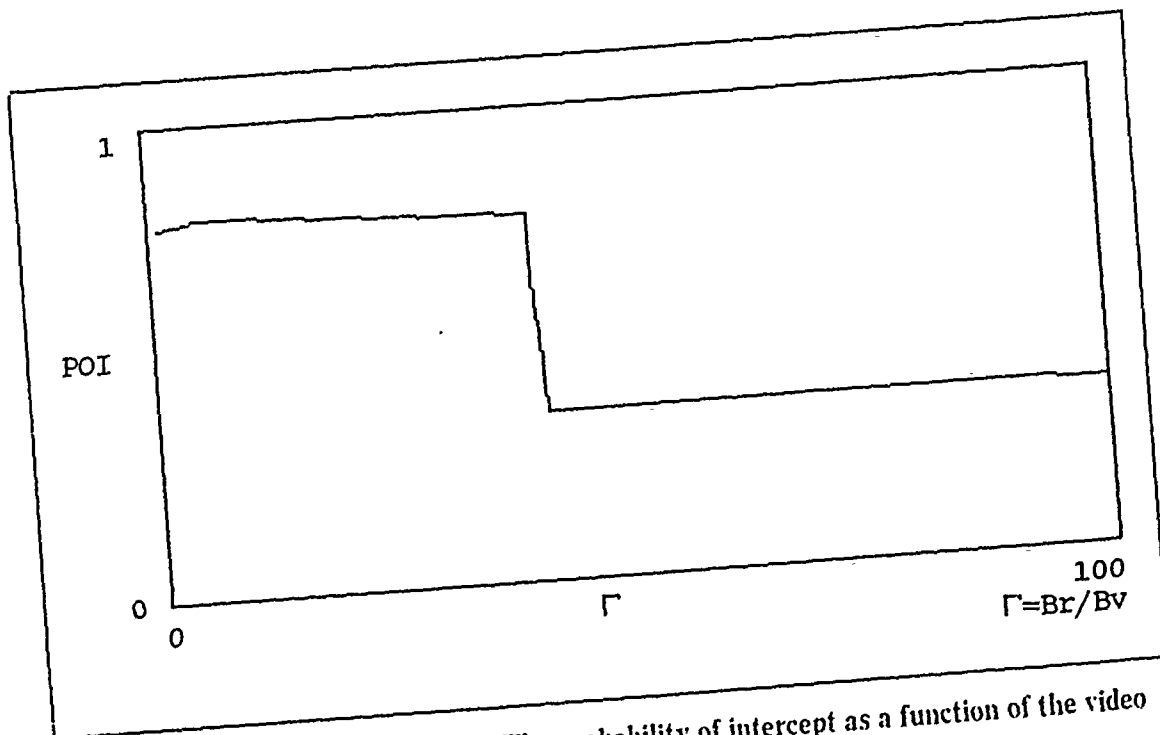

Figure 17. The POIG plot(2) - The probability of intercept as a function of the video bandwidth: $P_t = 15$ milliwatts $(SNR \approx 3dB)$
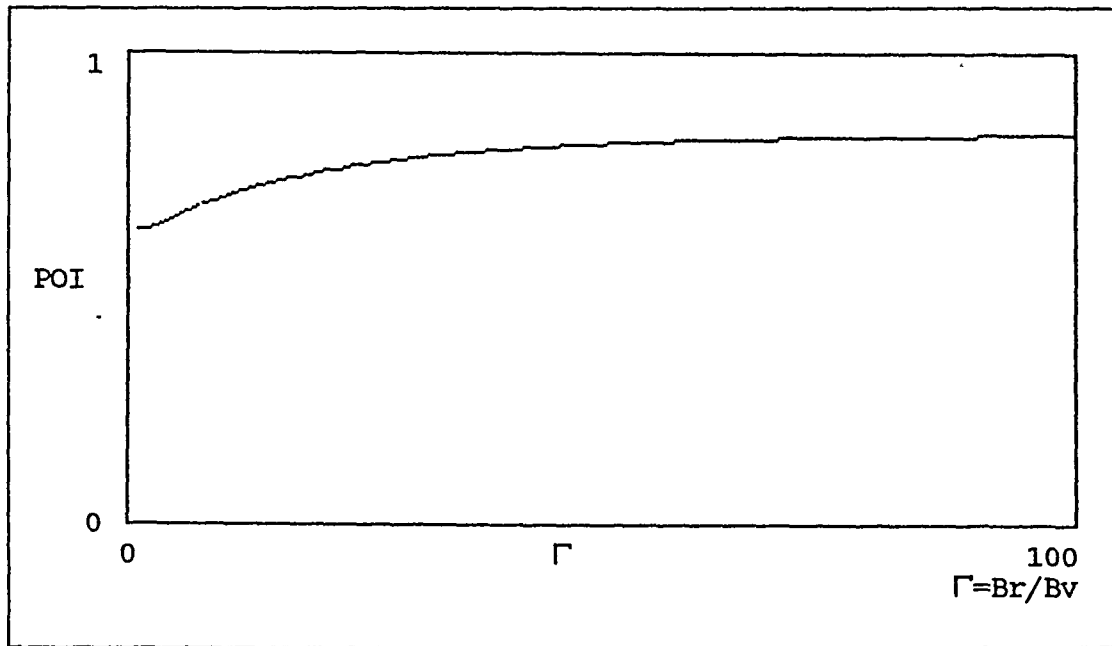
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Figure 18. The POIG plot(3) - The probability of intercept as a function of the video
bandwidth: $P_t = 20$ milliwatts ($SNR \approx 5dB$)

In Fig.18, POIG increases as y increases, as we expected, because with larger P_t, SNR is greater than 3 dB and the signal increases faster than the noise with increasing y.

In this model, we observe that if the SNR is less than 3 dB, when y increases, the POI decreases. When SNR is greater than 3 dB, POI increases, as the value of y increases.

E. PROBABILITY OF INTERCEPT FOR MULTIPLE SYSTEMS

In the previous chapter, the theory of the POI for the multiple systems was developed in terms of binomial characteristics when each interceptor has the same POI. In order to intercept a signal with the intercept system working in a dense, dynamic environment, multiple interceptors are necessary for a POI of unity. Certain assumptions are made prior to demonstrating this:

- Individual intercept system function independently
- Individual intercept system has the same POI

Consequently, the POI for N systems should have binomial characteristics. Fig.19 shows the results of the analysis.
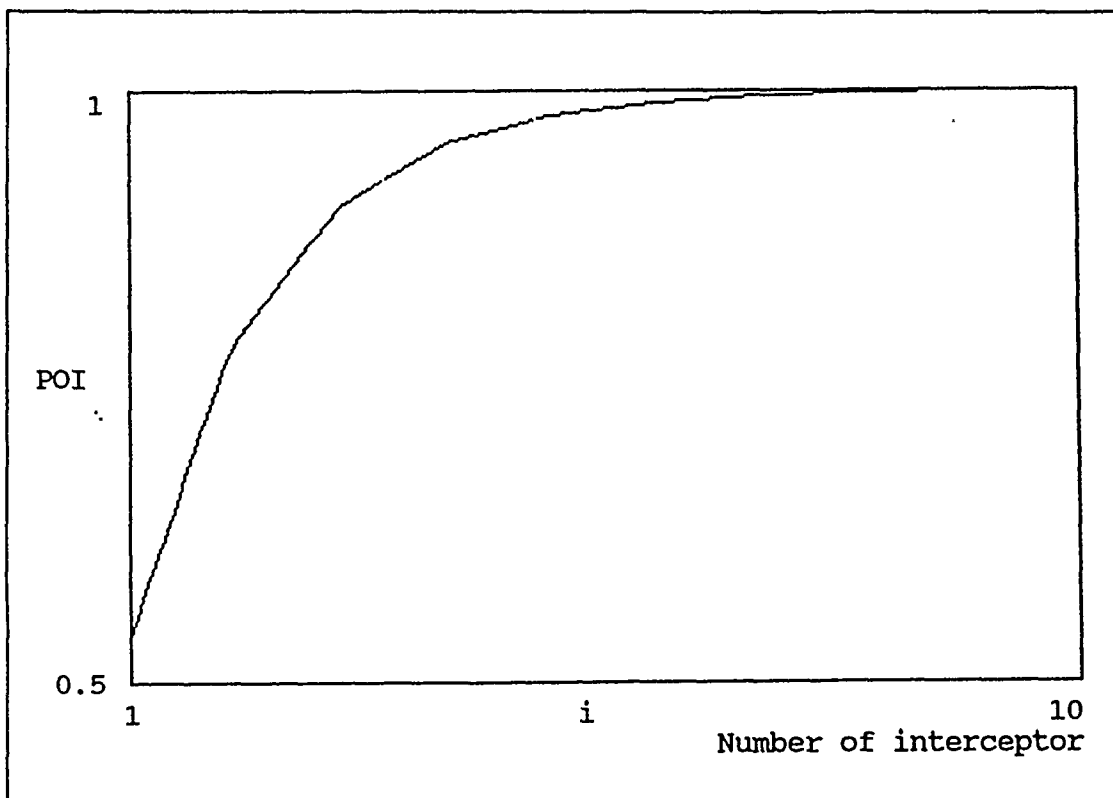
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Figure 19. The POIM plot: The probability of intercept for multiple systems

In this model, we observe that we need at least six intercept systems in order to achieve almost 100 % interception of the signal of interest.

V. CONCLUSION

In this thesis, we introduced the concepts and difficulties involved in calculation of the POI for the ground-based communication intercept systems. The POI can be estimated from the probability of detection and the probability of coincidence.

Probability of detection provides a measure of the receiver's capability to detect a signal in the presence of noise. Because of the complexities of the signal environment and the intercept receiver, the signal detection can not be determined in a deterministic way. However, the signal detection can be expressed probabilistically as a function of the SNR and the probability of false alarm. The SNR is a function of the various factors, which are the transmitted power, the antenna gain, the path loss, the receiver noise figure and the receiver sweeping loss. We reviewed these factors in Chapter Two. The probability of detection was derived from the equation developed by Skolnik.[Ref. 12]

If we have the very sensitive wideband receiver and the 360° coverage antenna, we may intercept the signal which cross the detection threshold. However, because of the cost and the high sensitivity of the receiver, we generally use the scanning superheterodyne receiver and the scanning antenna. In this case, even though the signal has sufficient strength to cross the detection threshold, it is not intercepted unless this scanning factor coincide each other. This is very likely to be the situation in COMINT operation.

The coincidence concept is introduced to model the main cause of the problem. Since the operating time of the emitter, and the scanning factors of the receiver and the antenna, behave stochastically and independently, we represent the probability of coincidence as a product of the probability of transmitter-on and the probability of observation. The probability of observation is mainly a function of the coincidence factor of the scanning parameters.

There are many previous works on the POI for radar ESM but not for COMINT. Since many of the basic concepts of radar ESM and COMINT are the same, we built a POI calculation model for COMINT by appling radar ESM concepts.

Since the electromagnetic activities in the atmosphere for the COMINT operation are somewhat similar to the radar ESM environment, the definition of the probability of detection can be applicable to COMINT analysis. Communication activities generally occur at HF and VHF range. Since this range is generally lower than the radar

frequency range, the propagation attenuation factor relating to the radio horizon was disscussed and analyzed. There are various parameters affecting the propagation factor. In this thesis, two types of the propagation were discussed; one is the free-space propagation within LOS, the other is the spread propagation beyond the radio horizon. Since we use a scanning superheterodyne type receiver, we discussed the sweeping loss The sweeping loss is a factor in reducing the SNR of the receiver output.

To apply the coincidence concept to the COMINT scenario, we addressed the problem where the signal duration is usually long enough to be intercepted by the scanning receiver. The signal does not have the periodic nature of radar. It makes sense that if scanning time is less than the signal duration, the signal should be intercepted. This is, however, not practical for a typical COMINT receiver. Therefore we tried an approach more suitable for available equipment performance specifications. We eliminated the signal window function from the coincidence calculation and introduced the concept of a joint occurence between two independent events which are; the event of transmitter-on and the event of observation.

The definition of POI was presented as the product of two independent probabilities, which are the probability of detection, the probability of coincidence. The probability of coincidence is defined as the product of the probability of transmitter-on and the probability of observation.

Since the probability of observation is a function of time, so is the POI In this discussion, the most consequential factor is the coincidence factor of the scanning parameters. If one has a unity coincidence factor, a wide bandwidth coverage receiver, and the transmitter operating all the time, we have unity POI. Otherwise, the interception of the signal is not guaranteed and time is required to obtain a given POI.

We discussed and analyzed multipath effects. The multipath factor is mainly a function of the separation distance, antenna heights and the frequency of the signal. As one increases the separation distance, one observes that the interference phenomena (either constructive or destructive) occur and the POI downgrades while it fluctuates. The POI behaves in the same pattern for the antenna elevations and the frequency. Also since the antenna height affects the radio horizon, above that distance, the POI is severely reduced.

We also discussed and analyzed bandwidth effects. We demonstrated that the receiver acceptance bandwidth can be optimized, since the probability of detection may actually decrease when the receiver acceptance bandwidth increases, while the probability of coincidence always increases. The video bandwidth was also discussed and

analyzed. It was found that above the 3 *dB* SNR point that the POI increases when the ratio of the RF bandwidth to the video bandwidth, y, increases. However, when the SNR is less than 3 *dB*, the POI decreases as y increases.

We then discussed the POI for multiple systems. Under a given probability of detection and the probability of coincidence, we demonstrated that the POI can reach near unity when we use the optimal number of intercept systems.

Since the coincidence concept is the most difficult to analyze mathematically, and represents the weakest part of the model developed in this thesis, it is recommended that further study on this concept be carried out. Because of the stochastic nature of the signal and the complexity of the analysis, it is reasonable to develop a Monte Carlo simulation program. The POI of COMINT systems is a function of the various parameters of the emitter and the receiver. Thus, POI can be described in terms of dynamic engagements of emitter/receiver parameters. This thesis is one approach to evaluate the capability of COMINT systems in a dynamic electronic warfare environment.

APPENDIX A. CHARACTERISTICS OF GROUND BASED COMMUNICATION EQUIPMENT

Following information is based on Jane's Defense Data [Ref. 19]

Table 3. GROUND BASED COMMUNICATION EQUIPMENT FOR HYPOTHETICAL HOSTILE FORCES

System	Band	Power Output (W)	Frequency (MHz)	Range (km)	Status
R-102M	HF	900	N/A	N/A	vehicular
R-103M	HF	50	N/A	N/A	vehicular
R-104,104M	HF	1/10	1.5-4.25	20-50	manpack or vehicular
R-105	HF	1	36.0-46.1	N/A	vehicular or manpack
R-108	HF	1	28.0-36.5	N/A	vehicular or manpack
R-109	HF	1	21.5-28.5	N/A	vehicular or manpack
R-114	HF	1	20-26	N/A	vehicular or manpack
R-106	VHF	0.5	48.65-51.35	2-3	manpack
R-107,107T	HF/VHF-FM	1	20-52	6-25	manpack
R-111	HF/VHF	75	20-52	35	vehicular
R-112	HF/AM	50-90	2.8-4.99	25-100	vehicular
R-113	HF/FM	16	20-22.375	20	vehicular
R-116	VHF	0.1	48.65-51.35	1	manpack
R-118,118BM	HF	250	1.0-7.5	600	vehicular

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Table 4. GROUND BASED COMMUNICATION EQUIPMENT FOR HY-POTHETICAL HOSTILE FORCES(CONT.)

R-123.123M	HF/VHF	20	20-51.5	20-50	vehicular
R-125	HF/VHF	N/A	N/A	N/A	vehicular
R-126	VHF/FM	0.5	48.5-51.0	1-2	manpack
R-130	HF	10-40	1.5-10.99	75/350	vehicular
R-148	VHF	1.1-2.1	37.0-51.95	5	manpack
R-154	HF	N/A	1.0-12.0	N/A	N/A
R-303	HF	13-24	3.024-22.832	N/A	vehicular
R-392A	VHF	1	44.0-46.1	N/A	manpack
R-401,403	VHF/FM	2.5	60-70	40-50	vehicular
R-405	UHF/FM	2.5	320-420	40-50	vehicular
R-1125F	HF/VHF	N/A	N/A	N/A	vehicular
5P21B-1	VHF	8-15	33-46	15-20	fixed
5P21C-3	VHF	8-15	33-46	N/A	fixed
Angara	HF/SSB	10/100	1.6-9	500	fixed
Mayak-S	M/C	12	146-174	15-30	fixed
PKM-5.20	HF ISB	5K/20K	3-30	N/A	fixed
Polyet-1A	VHF	5	100-149.975	N/A	fixed
Viola	UHF	8	148-173	N/A	fixed
YADRO-2	HF	400	2-30	N/A	fixed

APPENDIX B. ALGORITHM DERIVATION OF INTERCEPT PROBABILITY FOR MULTIPLE SYSTEMS

As disscussed in Chapter IV, we may calculate the probability of n simultaneous intercept sustems if the intercept probabilities of the individual intercept system are known. Following discussion is based on [Ref. 18]. Let

$$\bar{p}_{Ij} = (1 - p_{Ij}) \tag{B.1}$$

Then for $N = 2$ it follows that

$$P(0) = \bar{p}_{I1}\bar{p}_{I2} \tag{B.2a}$$

$$P(1) = (p_{I1}/\bar{p}_{I1} + p_{I2}/\bar{p}_{I2})\bar{p}_{I1}\bar{p}_{I2} \tag{B.2b}$$

$$P(2) = p_{I1}p_{I2} \tag{B.2c}$$

and similarly for $N = 3$ we obtain

$$P(0) = \prod_{j=1}^{3} \bar{p}_{Ij} \tag{B.3a}$$

$$P(1) = (p_{I1}/\bar{p}_{I1} + p_{I2}/\bar{p}_{I2} + p_{I3}/\bar{p}_{I3})\prod_{j=1}^{3} \bar{p}_{Ij} \tag{B.3b}$$

$$P(2) = (\bar{p}_{I1}/p_{I1} + \bar{p}_{I2}/p_{I2} + \bar{p}_{I3}/p_{I3})\prod_{j=1}^{3} \bar{p}_{Ij} \tag{B.3c}$$

$$P(3) = \prod_{j=1}^{3} p_{Ij} \tag{B.3d}$$

Now suppose we know $P(n)$ and wish to solve for p_{Ij}. Define

$$x_j = p_{Ij}/\bar{p}_{Ij} \tag{B.4}$$

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By algebraic substition among (B.2) or (B.3), we find

$$P(0)x_j^2 - P(1)x_j + P(2) = 0 \qquad (B.5)$$

for $N = 2$ and

$$P(0)x_j^3 - P(1)x_j^2 + P(2)x_j - P(3) = 0 \qquad (B.6)$$

for $N = 3$. In general, for any N the result is

$$P(0)x_j^N - P(1)x_j^{N-1} + P(2)x_j^{N-2} - \cdots + (-1)^N P(N) = 0 \qquad (B.7)$$

Thus the x_j are the roots of the polynomial equation(B.7) and from these roots we may calculate the p_{ij} using (B.1) through (B.4) as

$$p_{ij} = x_j/(1 + x_j) \qquad (B.8)$$

It is now clear if the p_{ij} are known we may obtain a polynomial having the form (B.7) as

$$\prod_{j=1}^{N}(x - x_j) = a_0 x^N + a_1 x^{N-1} + a_2 x^{N-2} + \cdots + a_N \qquad (B.9)$$

The coefficients a_n of this polynomial equation are proportional to $P(n)$. Knowing that

$$\sum_{n=0}^{N} P(n) = 1,$$

we recognize that

$$P(n) = |a_n| / \sum_{k=0}^{N} |a_k| \qquad (B.10)$$

This is the final form of the algorithm discussed in Chapter III.E.

APPENDIX C.   SETUP FILE

SETUP - This file initializes all emitter and interceptor system
parameters. Any change to these parameters will be read by
the probability files as they are excuted.
Therefore, after changing a parameter, ensure that any
files desired to change are loaded and excuted.

EMITTER AND INTERCEPTOR PARAMETERS

$PtGt := 1$ Effective radiated power of emitter (Watts)

$diam := 1$ Aperture diameter of the antenna (meters)

$freq := 50 \cdot 10^{6}$ Center frequency (hertz)

$dist := 20$ Distance between the emitter and the receiver
                             (kilometers)

$ht := 150$ Emitter antenna elevation above sea level
                             (meters)

$hr := 150$ Intercept antenna elevation above sea level
                             (meters)

$Br := 10 \cdot 10^{6}$ Receiver acceptance bandwidth (hertz)

$FndB := 10$ Noise figure   (dB)

$Ds := 500 \cdot 10^{6}$ Total frequency coverage (hertz)

$Pfa := 1 \cdot 10^{-4}$ Probability of false alarm

$t_{on} := 5$ Average operating time of emitter (sec)

$t_{off} := 100$ Average off time of emitter (sec)

$t_{s} := .02$ Dwell time of the receiver (sec)

$T_{s} := 1$ Total scan time of the receiver (sec)

$t_{a} := 0.1$ Look dwell time of the antenna (sec)

$T_{a} := 1$ Total scan time of the antenna (sec)

$K := 5$ Correction factor

All parameters are written into the file called SETUP.PRN and wiil be called by the probability file when necessary

$i := 1 .. 17$

setup$_i$:=

PtGt
diam
freq
dist
ht
hr
Br
FndB
Ds
Pfa
t_{on}
t_{off}
t_s
T_s
t_a
T_a
K

$$\text{WRITEPRN}\begin{bmatrix} \text{setup}_{prn} \end{bmatrix} := \text{setup}_i$$

APPENDIX D.   MATHCAD WORKSHEET FOR INTERCEPT TIME
(POIT)

PROBABILITY OF INTERCEPT AS A FUNCTION OF TIME
==

POIT - This file calculates the probability of intercept as a
    function of time for a moving or fixed emitter as observed
    by a stationary scanning superhet receiver and a scaning
    antenna

$$data := READPRN \begin{bmatrix} setup \\ \quad prn \end{bmatrix}$$

Parameter definition
==============================

$PtGt := data_0$

$diam := data_1$

$freq := data_2$

$dist := data_3$

$ht := data_4$

$hr := data_5$

$Br := data_6$

$FndB := data_7$

$Ds := data_8$

$Pfa := data_9$

$ton := data_{10}$

$toff := data_{11}$

$ts := data_{12}$

$Ts := data_{13}$

$ta := data_{14}$

$Ta := data_{15}$

$K := data_{16}$

Received signal power calculation

$$PtGtdBm := 30 + 10 \cdot \log(PtGt)$$

$$\lambda := \frac{3 \cdot 10^8}{freq}$$

$$Gr := 0.54 \cdot \left[\frac{\pi \cdot diam}{\lambda}\right]^2$$

$$GrdB := 10 \cdot \log(Gr)$$

calculate the free space path loss

$$Lp_{fs} := 32.45 + 20 \cdot \log\left[\frac{freq}{10^6}\right] + 20 \cdot \log(dist)$$

calculate the path loss for beyond horizon distance

$$Lp_{sp} := 108 + 20 \cdot \log\left[\frac{freq}{10^6}\right] + 40 \cdot \log(dist) - 20 \cdot \log(ht \cdot hr) + 12$$

calculate the multipath factor

$$g2 := 2 \cdot \left| \sin\left[2 \cdot \pi \cdot \left[\frac{ht \cdot hr}{dist \cdot \lambda}\right]\right] \right|$$

calculate the received signal power for the free space

$$Si_{fs} := PtGtdBm + GrdB + 10 \cdot \log(g2) - Lp_{fs}$$

calculate the received signal power for the distance beyond the horizon

$$Si_{sp} := PtGtdBm + GrdB + 10 \cdot \log(g2) - Lp_{sp}$$

Noise power calculation

$$NidBm := -114 + 10 \cdot \log\left[\frac{Br}{10^6}\right]$$

Sweeping loss calculation
==========================

$$LsdB := \left[1 + 0.195 \cdot \left[\frac{Ds}{Ts \cdot Br^2}\right]^2\right]^{-0.25}$$

Signal to noise ratio calculation
====================================

signal to noise ratio for the free space

$$SNRdB_{fs} := Si_{fs} - NidBm - LsdB - FndB$$

$$SNR_{fs} := 10^{\frac{SNRdB_{fs}}{10}}$$

signal to noise ratio for the distance beyond the horizon

$$SNRdB_{sp} := Si_{sp} - NidBm - LsdB - FndB$$

$$SNR_{sp} := 10^{\frac{SNRdB_{sp}}{10}}$$

calculate the radio horizon distance

$$RHZ := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr}\right]$$

calculate the signal to noise ratio

$$SNR := \Phi(RHZ - dist) \cdot SNR_{fs} + \Phi(dist - RHZ) \cdot SNR_{sp}$$

PROBABILITY OF DETECTION CALCULATION

$$A := \sqrt{\ln\left[\frac{1}{Pfa}\right]} \qquad\qquad B := \sqrt{SNR}$$

$$Pd := \frac{1}{2}\cdot(1 - erf(A - B)) \quad \ldots$$
$$+ \frac{\exp\left[-(A - B)^2\right]}{4\cdot\sqrt{\pi}\cdot B}\cdot\left[0.75 - \frac{A}{4\cdot B} + \frac{1 + 2\cdot(A - B)^2}{16\cdot B^2}\right]$$

PROBABILITY OF COINCIDENCE AS A FUNCTION OF TIME CALCULATION

$$\Gamma := K\cdot\frac{ta\cdot ts}{Ta\cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$k := 1 \,..500$$

$$t_k := k$$

$$Pob_k := 1 - \exp\left[-\Gamma\cdot t_k\right]$$

$$Pc_k := Pon\cdot Pob_k$$

PROBABILITY OF INTERCEPT AS A FUNCTION OF TIME CALCULATION

$$POI_k := Pd \cdot Pc_k$$
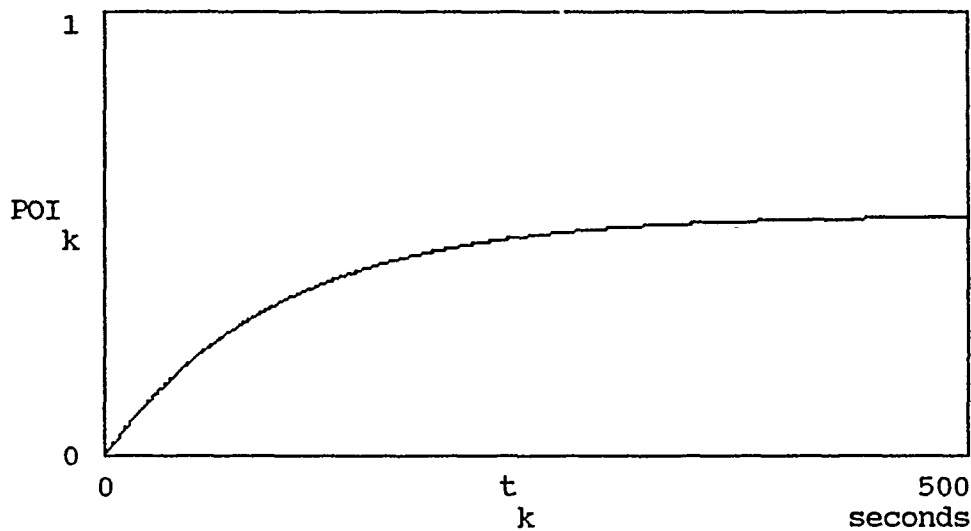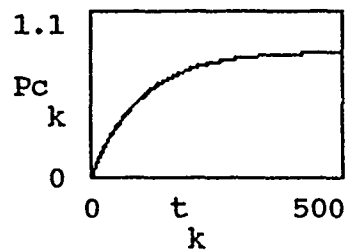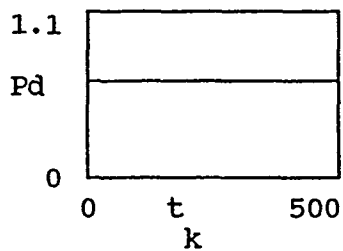
APPENDIX E.   MATHCAD WORKSHEET FOR MULTIPATH EFFECT

A. POID WORKSHEET

PROBABILITY OF INTERCEPT AS A FUNCTION OF DISTANCE
===

POIR -- This file calculates the probability of intercept as a
function of the separation distance between the emitter
and the intercept station as observed by a scanning
superhet receiver and a scaning antenna

$$data := READPRN \begin{bmatrix} setup \\ \quad prn \end{bmatrix}$$

Parameter definition
====================

$PtGt := data_0$

$diam := data_1$

$freq := data_2$

$i := 1 .. 100$

$dist_i := .5 \cdot i$

$ht := 10$

$hr := 10$

$Br := data_6$

$FndB := data_7$

$Ds := data_8$

$Pfa := data_9$

$ton := data_{10}$

$toff := data_{11}$

$ts := data_{12}$

$Ts := data_{13}$

$ta := data_{14}$

$Ta := data_{15}$

$K := data_{16}$

Received signal power calculation

$$PtGtdBm := 30 + 10 \cdot \log(PtGt)$$

$$\lambda := \frac{3 \cdot 10^8}{freq}$$

$$Gr := 0.54 \cdot \left[\frac{\pi \cdot diam}{\lambda}\right]^2$$

$$GrdB := 10 \cdot \log(Gr)$$

$$Lp_{fs_i} := 32.45 + 20 \cdot \log\left[\frac{freq}{10^6}\right] + 20 \cdot \log\left[dist_i\right]$$

$$Lp_{sp_i} := 108 + 20 \cdot \log\left[\frac{freq}{10^6}\right] + 40 \cdot \log\left[dist_i\right] - 20 \cdot \log(ht \cdot hr) + 12$$

$$g2_i := 2 \cdot \left|\sin\left[2 \cdot \pi \cdot \left[\frac{ht \cdot hr}{dist_i \cdot \lambda}\right]\right]\right|$$

$$Si_{fs_i} := PtGtdBm + GrdB - Lp_{fs_i}$$

$$Si_{sp_i} := PtGtdBm + GrdB - Lp_{sp_i}$$

Noise power calculation

$$NidBm := -114 + 10 \cdot \log\left[\frac{Br}{10^6}\right]$$

Sweeping loss calculation

$$LsdB := \left[1 + 0.195 \cdot \left[\frac{Ds}{Ts \cdot Br^2}\right]^2\right]^{-0.25}$$

Signal to noise ratio calculation
================================

$$\text{SNRdB}_{fs_i} := \text{Si}_{fs_i} - \text{NidBm} - \text{LsdB} - \text{FndB}$$

$$\text{SNR}_{fs_i} := 10^{\dfrac{\text{SNRdB}_{fs_i}}{10}} \cdot g2_i$$

$$\text{SNRdB}_{sp_i} := \text{Si}_{sp_i} - \text{NidBm} - \text{LsdB} - \text{FndB}$$

$$\text{SNR}_{sp_i} := 10^{\dfrac{\text{SNRdB}_{sp_i}}{10}} \cdot g2_i$$

Radio horizon calculation
=========================

$$\text{RHZ} := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr} \right]$$

$$\text{SNR}_i := \Phi\left[\text{RHZ} - \text{dist}_i \right] \cdot \text{SNR}_{fs_i} + \Phi\left[\text{dist}_i - \text{RHZ} \right] \cdot \text{SNR}_{sp_i}$$

PROBABILITY OF DETECTION CALCULATION
====================================

$$A := \sqrt{\ln\left[\dfrac{1}{\text{Pfa}} \right]} \qquad\qquad B_i := \sqrt{\text{SNR}_i}$$

$$Pd_i := \frac{1}{2} \cdot \left[1 - \text{erf}\left[A - B_i \right] \right] \ldots$$

$$+ \frac{\exp\left[-\left[A - B_i \right]^2 \right]}{4 \cdot \sqrt{\pi} \cdot B_i} \cdot \left[0.75 - \frac{A}{4 \cdot B_i} + \frac{1 + 2 \cdot \left[A - B_i \right]^2}{16 \cdot B_i^2} \right]$$

$$Pd_i := Pd_i \cdot \Phi\left[\text{SNR}_i - .00001 \right]$$

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Probability of coincidence calculation

======================================

$$\Gamma := K \cdot \frac{ta \cdot ts}{Ta \cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$t := 500$$

$$Pob := 1 - \exp(-\Gamma \cdot t)$$

$$Pc := Pon \cdot Pob$$

PROBABILITY OF INTERCEPT AS A FUNCTION OF DISTANCE CALCULATION

==

$$POI_i := Pd_i \cdot Pc_i$$
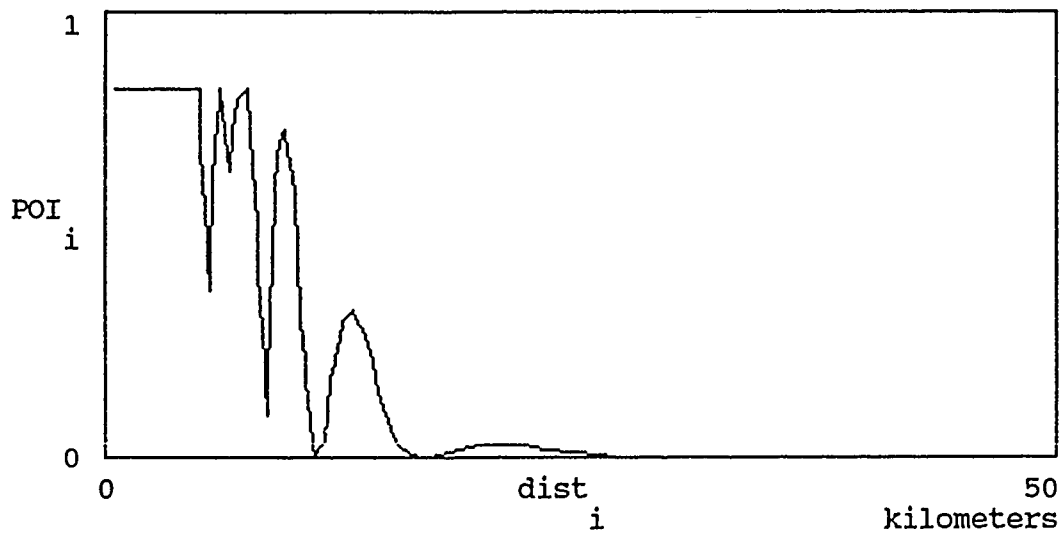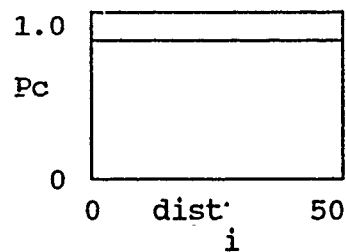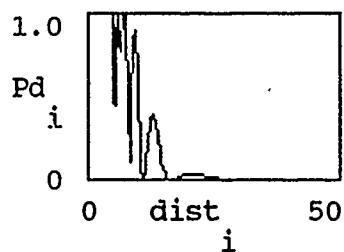
B. POIF WORKSHEET

PROBABILITY OF INTERCEPT AS A FUNCTION OF FREQUENCY

POIF - This file calculates the probability of intercept as a function of frequency for fixed or moving emitter as observed by a stationary scanning superhet receiver and a scaning antenna

$$data := READPRN \begin{bmatrix} setup \\ prn \end{bmatrix}$$

Parameter definition

$PtGt := data_0$

$diam := data_1$

$i := 1 ..100$

$freq_i := i \cdot 10^6$ Frequency range from 1 MHz to 500 MHz

$dis_c := data_3$

$ht := data_4$

$hr := data_5$

$Br := data_6$

$FndB := data_7$

$Ds := data_8$

$Pfa := data_9$

$ton := data_{10}$

$toff := data_{11}$

$ts := data_{12}$

$Ts := data_{13}$

$ta := data_{14}$

$Ta := data_{15}$

$K := data_{16}$

Received signal power calculation

$$PtGtdBm := 30 + 10 \cdot \log(PtGt)$$

$$\lambda_i := \frac{3 \cdot 10^8}{freq_i}$$

$$Gr_i := 0.54 \cdot \left[\frac{\pi \cdot diam}{\lambda_i}\right]^2$$

$$GrdB_i := 10 \cdot \log\left[Gr_i\right]$$

$$Lp_{fs_i} := 32.45 + 20 \cdot \log\left[\frac{freq_i}{10^6}\right] + 20 \cdot \log(dist)$$

$$Lp_{sp_i} := 108 + 20 \cdot \log\left[\frac{freq_i}{10^6}\right] + 40 \cdot \log(dist) - 20 \cdot \log(ht \cdot hr) + 12$$

$$g2_i := 2 \cdot \left|sin\left[2 \cdot \pi \cdot \left[\frac{ht \cdot hr}{dist \cdot \lambda_i}\right]\right]\right|$$

$$Si_{fs_i} := PtGtdBm + GrdB_i - Lp_{fs_i}$$

$$Si_{sp_i} := PtGtdBm + GrdB_i - Lp_{sp_i}$$

Noise power calculation

$$NidBm := -114 + 10 \cdot \log\left[\frac{Br}{10^6}\right]$$

Sweeping loss calculation

$$LsdB := \left[1 + 0.195 \cdot \left[\frac{Ds}{Ts \cdot Br^2}\right]^2\right]^{-0.25}$$

Signal to noise ratio calculation
=====================================

$$SNRdB_{fs_i} := Si_{fs_i} - NidBm - LsdB - FndB$$

$$SNR_{fs_i} := 10^{\dfrac{SNRdB_{fs_i}}{10}} \cdot g2_i$$

$$SNRdB_{sp_i} := Si_{sp_i} - NidBm - LsdB - FndB$$

$$SNR_{sp_i} := 10^{\dfrac{SNRdB_{sp_i}}{10}} \cdot g2_i$$

Radio horizon calculation
==========================

$$RHZ := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr}\right]$$

$$SNR_i := \Phi(RHZ - dist) \cdot SNR_{fs_i} + \Phi(dist - RHZ) \cdot SNR_{sp_i}$$

PROBABILITY OF DETECTION CALCULATION
======================================

$$A := \sqrt{\ln\left[\dfrac{1}{Pfa}\right]} \qquad\qquad B_i := \sqrt{SNR_i}$$

$$Pd_i := \dfrac{1}{2} \cdot \left[1 - erf\left[A - B_i\right]\right] \cdots$$

$$+ \dfrac{\exp\left[-\left[A - B_i\right]^2\right]}{4 \cdot \sqrt{\pi} \cdot B_i} \cdot \left[0.75 - \dfrac{A}{4 \cdot B_i} + \dfrac{1 + 2 \cdot \left[A - B_i\right]^2}{16 \cdot B_i^{\,2}}\right]$$

$$Pd_i := Pd_i \cdot \Phi\left[SNR_i - .00001\right]$$

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Probability of coincidence calculation

$$\Gamma := K \cdot \frac{ta \cdot ts}{Ta \cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$t := 500$$

$$Pob := 1 - \exp(-\Gamma \cdot t)$$

$$Pc := Pon \cdot Pob$$

PROBABILITY OF INTERCEPT AS A FUNCTION OF FREQUENCY CALCULATION

$$POI_i := Pd_i \cdot Pc$$
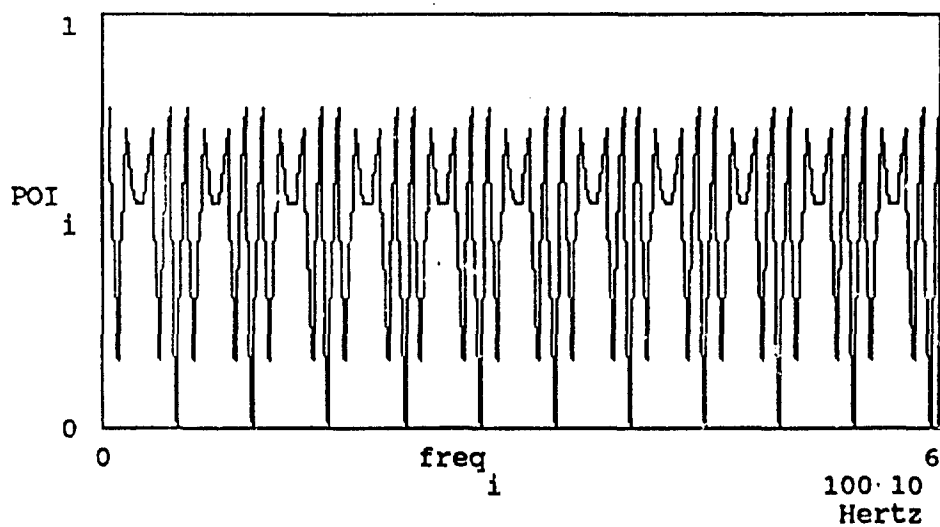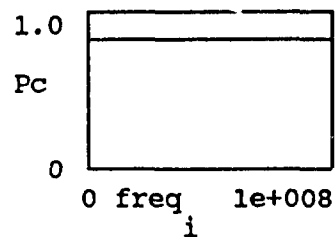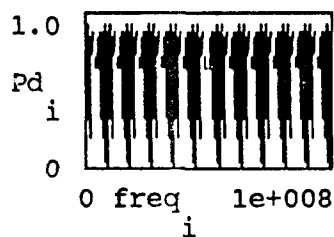
C. POIA WORKSHEET

PROBABILITY OF INTERCEPT AS A FUNCTION OF ANTENNA ELEVATION

POIA - This file calculates the probability of intercept as a
function of interceptor antenna height/elevation for a
moving or fixed emitter as observed by a stationary
scanning superhet receiver and a scaning antenna

$$data := READPRN\left[\begin{matrix} setup \\ \quad prn \end{matrix}\right]$$

Parameter definition

$PtGt := data_0$

$diam := data_1$

$freq := data_2$

$i := 1 ..150$

$dist := data_3$

$ht := data_4$

$hr_i := 150 + i$ Interceptor antenna height/elevation
from 151 meters to 300 meters above
$Br := data_6$ the sea level

$FndB := data_7$

$Ds := data_8$

$Pfa := data_9$

$ton := data_{10}$

$toff := data_{11}$

$ts := data_{12}$

$Ts := data_{13}$

$ta := data_{14}$

$Ta := data_{15}$

$K := data_{16}$

Received signal power calculation

$$PtGtdBm := 30 + 10 \cdot \log(PtGt)$$

$$\lambda := \frac{3 \cdot 10^8}{freq}$$

$$Gr := 0.54 \cdot \left[\frac{\pi \cdot diam}{\lambda}\right]^2$$

$$GrdB := 10 \cdot \log(Gr)$$

$$Lp_{fs} := 32.45 + 20 \cdot \log\left[\frac{freq}{10^6}\right] + 20 \cdot \log(dist)$$

$$Lp_{sp_i} := 108 + 20 \cdot \log\left[\frac{freq}{10^6}\right] + 40 \cdot \log(dist) - 20 \cdot \log\left[ht \cdot hr_i\right] + 12$$

$$g2_i := 2 \cdot \left| \sin\left[2 \cdot \pi \cdot \left[\frac{ht \cdot hr_i}{dist \cdot \lambda}\right]\right]\right|$$

$$Si_{fs} := PtGtdBm + GrdB - Lp_{fs}$$

$$Si_{sp_i} := PtGtdBm + GrdB - Lp_{sp_i}$$

Noise power calculation

$$NidBm := -114 + 10 \cdot \log\left[\frac{Br}{10^6}\right]$$

Sweeping loss calculation

$$LsdB := \left[1 + 0.195 \cdot \left[\frac{Ds}{Ts \cdot Br^2}\right]^2\right]^{-0.25}$$

Signal to noise ratio calculation
================================

$$SNRdB_{fs} := Si_{fs} - NidBm - LsdB - FndB$$

$$SNR_{fs_i} := 10^{\dfrac{SNRdB_{fs}}{10}} \cdot g2_i$$

$$SNRdB_{sp_i} := Si_{sp_i} - NidBm - LsdB - FndB$$

$$SNR_{sp_i} := 10^{\dfrac{SNRdB_{sp_i}}{10}} \cdot g2_i$$

Radio horizon calculation
================================

$$RHZ_i := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr_i} \right]$$

$$SNR_i := \Phi\left[RHZ_i - dist \right] \cdot SNR_{fs_i} + \Phi\left[dist - RHZ_i \right] \cdot SNR_{sp_i}$$

PROBABILITY OF DETECTION CALCULATION
================================

$$A := \sqrt{\ln\left[\dfrac{1}{Pfa}\right]} \qquad\qquad B_i := \sqrt{SNR_i}$$

$$Pd_i := \dfrac{1}{2}\left[1 - erf\left[A - B_i\right]\right] \ldots$$

$$+ \dfrac{\exp\left[-\left[A - B_i\right]^2\right]}{4\cdot\sqrt{\pi}\cdot B_i} \cdot \left[0.75 - \dfrac{A}{4\cdot B_i} + \dfrac{1 + 2\cdot\left[A - B_i\right]^2}{16\cdot B_i^2}\right]$$

$$Pd_i := Pd_i \cdot \Phi\left[SNR_i - .00001\right]$$

Probability of coincidence calculation
==

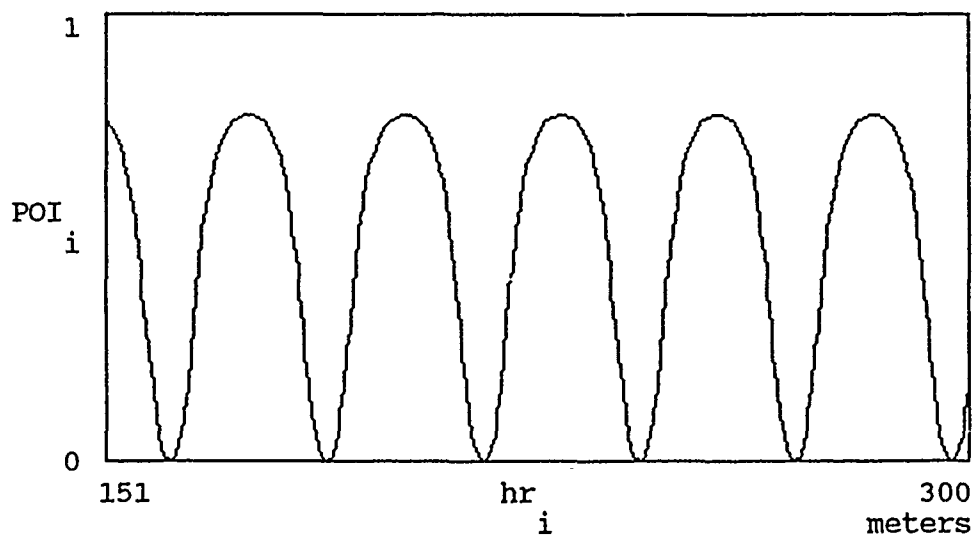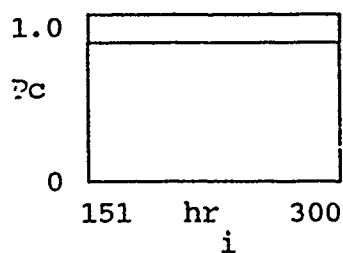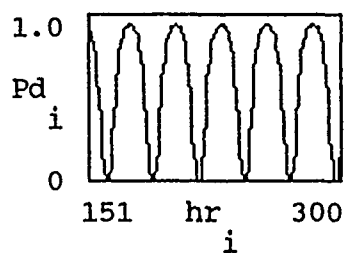$$\Gamma := K \cdot \frac{ta \cdot ts}{Ta \cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$t := 500$$

$$Pob := 1 - \exp(-\Gamma \cdot t)$$

$$Pc := Pon \cdot Pob$$

PROBABILITY OF INTERCEPT AS A FUNCTION OF ANTENNA ELEVATION
==

$$POI_i := Pd_i \cdot Pc_i$$

APPENDIX F. MATHCAD WORKSHEET FOR BANDWIDTH EFFECT

A. POIB WORKSHEET

POIR - This file calculates the probability of intercept as a function of the receiver acceptance bandwidth for a moving or fixed emitter as observed by a stationary scanning superhet receiver and a scaning antenna

$$data := READPRN \begin{bmatrix} setup \\ prn \end{bmatrix}$$

Parameter definition
====================

$PtGt := data_0$

$diam := data_1$

$freq := data_2$

$i := 1 ..100$

$dist := data_3$

$ht := data_4$

$hr := data_5$

$Br_i := i \cdot 10^6$

$FndB := data_7$

$Ds := data_8$

$Pfa := data_9$

$ton := data_{10}$

$toff := data_{11}$

$Ts := data_{13}$

$ts_i := Ts \cdot \dfrac{Br_i}{Ds}$

$ta := data_{14}$

$Ta := data_{15}$

$K := data_{16}$

Received signal power calculation
===============================

$PtGtdBm := 30 + 10 \cdot \log(PtGt)$

$\lambda := \dfrac{3 \cdot 10^8}{freq}$

$Gr := 0.54 \cdot \left[\dfrac{\pi \cdot diam}{\lambda}\right]^2$

$GrdB := 10 \cdot \log(Gr)$

$Lp_{fs} := 32.45 + 20 \cdot \log\left[\dfrac{freq}{10^6}\right] + 20 \cdot \log(dist)$

$Lp_{sp} := 108 + 20 \cdot \log\left[\dfrac{freq}{10^6}\right] + 40 \cdot \log(dist) - 20 \cdot \log(ht \cdot hr) + 12$

$g2 := 2 \cdot \left| \sin\left[2 \cdot \pi \cdot \left[\dfrac{ht \cdot hr}{dist \cdot \lambda}\right]\right] \right|$

$Si_{fs} := PtGtdBm + GrdB - Lp_{fs}$

$Si_{sp} := PtGtdBm + GrdB - Lp_{sp}$

Noise power calculation
======================

$NidBm_i := -114 + 10 \cdot \log\left[\dfrac{Br_i}{10^6}\right]$

Sweeping loss calculation
========================

$LsdB_i := \left[1 + 0.195 \cdot \left[\dfrac{Ds}{Ts \cdot Br_i^2}\right]^2\right]^{-0.25}$

Signal to noise ratio calculation

$$SNRdB_{fs_i} := Si_{fs} - NidBm_i - LsdB_i - FndB$$

$$SNR_{fs_i} := 10^{\dfrac{SNRdB_{fs_i}}{10}} \cdot g2$$

$$SNRdB_{sp_i} := Si_{sp} - NidBm_i - LsdB_i - FndB$$

$$SNR_{sp_i} := 10^{\dfrac{SNRdB_{sp_i}}{10}} \cdot g2$$

Radio horizon calculation

$$RHZ := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr} \right]$$

$$SNR := \Phi(RHZ - dist) \cdot SNR_{fs} + \Phi(dist - RHZ) \cdot SNR_{sp}$$

PROBABILITY OF DETECTION CALCULATION

$$A := \sqrt{\ln\left[\dfrac{1}{Pfa}\right]} \qquad\qquad B_i := \sqrt{SNR_i}$$

$$Pd_i := \dfrac{1}{2} \cdot \left[1 - erf\left[A - B_i \right]\right] \ldots$$

$$+ \dfrac{\exp\left[-\left[A - B_i\right]^2 \right]}{4 \cdot \sqrt{\pi} \cdot B_i} \cdot \left[0.75 - \dfrac{A}{4 \cdot B_i} + \dfrac{1 + 2 \cdot \left[A - B_i\right]^2}{16 \cdot B_i^2} \right]$$

$$Pd_i := Pd_i \cdot \Phi\left[SNR_i - .00001 \right]$$

PROBABILITY OF COINCIDENCE CALCULATION

$$\Gamma_i := K \cdot \frac{ta \cdot ts_i}{Ta \cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$t := 500$$
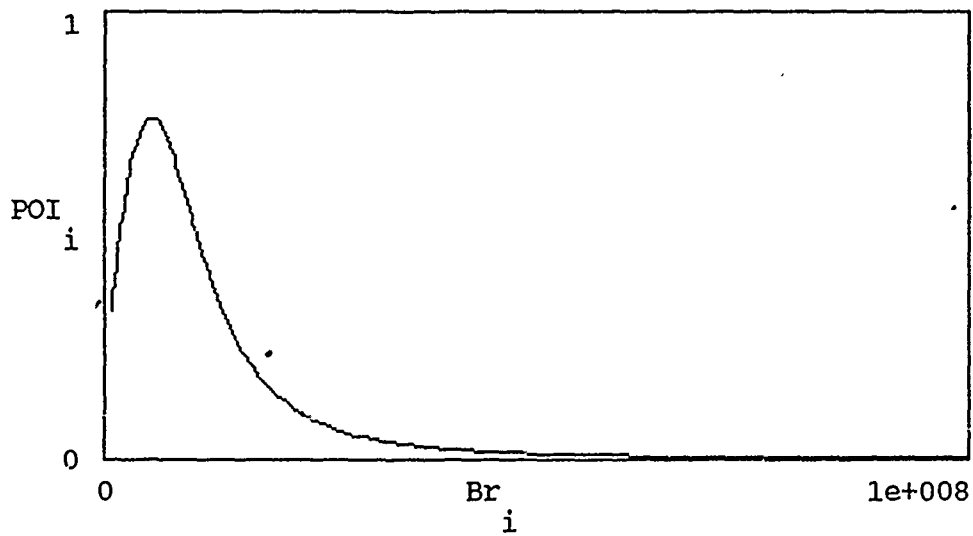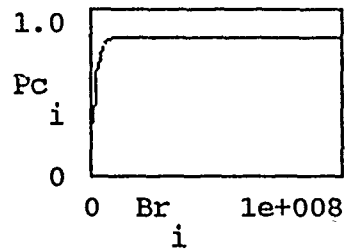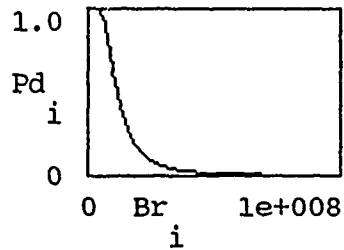
$$Pob_i := 1 - \exp\left[-\Gamma_i \cdot t\right]$$

$$Pc_i := Pon \cdot Pob_i$$

PROBABILITY OF INTERCEPT CALCULATION

$$POI_i := Pd_i \cdot Pc_i$$

B. POIV WORKSHEET

PROBABILITY OF INTERCEPT VERSUS VIDEO BANDWIDTH FILE
==

POIV - This file calculates the probability of intercept as a
function of the ratio of the RF bandwidth to the video
bandwidth for a moving or fixed emitter as observed by
a stationary scanning general type receiver and a scaning
antenna

$$\text{data} := \text{READPRN} \begin{bmatrix} \text{setup} \\ \text{prn} \end{bmatrix}$$

Parameter definition
======================

$PtGt := \text{data}_0$

$diam := \text{data}_1$

$freq := \text{data}_2$

$dist := \text{data}_3$

$ht := \text{data}_4$

$hr := \text{data}_5$

$Br := \text{data}_6$

$FndB := \text{data}_7$

$Ds := \text{data}_8$

$Pfa := \text{data}_9$

$ton := \text{data}_{10}$

$toff := \text{data}_{11}$

$ts := \text{data}_{12}$

$Ts := \text{data}_{13}$

$ta := \text{data}_{14}$

$Ta := \text{data}_{15}$

$K := \text{data}_{16}$

Received signal power calculation

$$PtGtdBm := 30 + 10 \cdot \log(PtGt)$$

$$\lambda := \frac{3 \cdot 10^{8}}{freq}$$

$$Gr := 0.54 \cdot \left[\frac{\pi \cdot diam}{\lambda}\right]^{2}$$

$$GrdB := 10 \cdot \log(Gr)$$

$$Lp_{fs} := 32.45 + 20 \cdot \log\left[\frac{freq}{10^{6}}\right] + 20 \cdot \log(dist)$$

$$Lp_{sp} := 108 + 20 \cdot \log\left[\frac{freq}{10^{6}}\right] + 40 \cdot \log(dist) - 20 \cdot \log(ht \cdot hr) + 12$$

$$g2 := 2 \cdot \left|\sin\left[2 \cdot \pi \cdot \left[\frac{ht \cdot hr}{dist \cdot \lambda}\right]\right]\right|$$

$$Si_{fs} := PtGtdBm + GrdB + 10 \cdot \log(g2) - Lp_{fs}$$

$$Si_{sp} := PtGtdBm + GrdB + 10 \cdot \log(g2) - Lp_{sp}$$

Noise power calculation

$$NidBm := -114 + 10 \cdot \log\left[\frac{Br}{10^{6}}\right]$$

Sweeping loss calculation

$$LsdB := \left[1 + 0.195 \cdot \left[\frac{Ds}{Ts \cdot Br^{2}}\right]^{2}\right]^{-0.25}$$

Signal to noise ratio calculation

$$SNRdB_{fs} := Si_{fs} - NidBm - LsdB - FndB$$

$$SNR_{fs} := 10^{\dfrac{SNRdB_{fs}}{10}}$$

$$SNRdB_{sp} := Si_{sp} - NidBm - LsdB - FndB$$

$$SNR_{sp} := 10^{\dfrac{SNRdB_{sp}}{10}}$$

Radio horizon calculation

$$RHZ := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr} \right]$$

$$SNR := \Phi(RHZ - dist) \cdot SNR_{fs} + \Phi(dist - RHZ) \cdot SNR_{sp}$$

PROBABILITY OF DETECTION CALCULATION

$$i := 1 \ ..100$$

$$\Gamma_i := i \qquad VT := \sqrt{\ln\left[\dfrac{1}{Pfa}\right]}$$

$$K1 := 1 + SNR$$

$$K2_i := \dfrac{1}{\left[1 + \dfrac{\Gamma_i^2}{2}\right]^{.5}} \cdot \left[1 + SNR \cdot \left[\dfrac{1 + \dfrac{\Gamma_i^2}{2}}{1 + \dfrac{\Gamma_i^2}{4}} \right] \right]$$

$$K5_i := \dfrac{VT - K1}{\sqrt{K2_i}}$$

$$K3_i := \frac{4}{2 + 3 \cdot \frac{\Gamma_i^2}{4}} \cdot \left[1 + 3 \cdot SNR \cdot \left[\frac{2 + 3 \cdot \frac{\Gamma_i^2}{4}}{2 + \frac{\Gamma_i^2}{4}}\right]^{.5}\right] \qquad\qquad K4_i := \frac{K1^2}{K2_i}$$

$$A_i := \frac{1}{\sqrt{2 \cdot \pi}} \cdot \frac{K3_i}{6 \cdot K2_i^{1.5}} \cdot \left[K4_i - 1\right] \cdot \exp\left[\frac{-K4_i}{2}\right] + \left[\frac{1}{2} - \frac{1}{2} \cdot erf\left[-\sqrt{\frac{K4_i}{2}}\right]\right]$$

$$B_i := \frac{1}{\sqrt{2 \cdot \pi}} \cdot \frac{K3_i}{6 \cdot K2_i^{1.5}} \cdot \left[K5_i^2 - 1\right] \cdot \exp\left[\frac{-K5_i^2}{2}\right] + \left[\frac{1}{2} - \frac{1}{2} \cdot erf\left[\frac{K5_i}{\sqrt{2}}\right]\right]$$

$$C_i := \frac{1}{\sqrt{2 \cdot \pi}} \cdot \frac{K3_i}{6 \cdot K2_i^{1.5}} \cdot \left[K5_i^2 - 1\right] \cdot \exp\left[\frac{-K5_i^2}{2}\right] \qquad \cdots$$

$$+ \frac{1}{\sqrt{2 \cdot \pi}} \cdot \frac{K3_i}{6 \cdot K2_i^{1.5}} \cdot \left[K4_i - 1\right] \cdot \exp\left[\frac{-K4_i}{2}\right] \qquad \cdots$$

$$+ \left[\frac{1}{2} \cdot erf\left[\frac{K5_i}{\sqrt{2}}\right] - \frac{1}{2} \cdot erf\left[-\sqrt{\frac{K4_i}{2}}\right]\right]$$

$$Pd1_i := \frac{B_i}{A_i} \qquad\qquad Pd2_i := \frac{A_i - C_i}{A_i}$$

$$Pd_i := Pd1_i \cdot \Phi\left[.5 - Pd1_i\right] + Pd2_i \cdot \Phi\left[Pd2_i - .5\right]$$

Probability of coincidence calculation
=======================================

$$\tau := K \cdot \frac{ta \cdot ts}{Ta \cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$t := 500$$

$$Pob := 1 - \exp(-\tau \cdot t)$$

$$Pc := Pon \cdot Pob$$

PROBABILITY OF INTERCEPT CALCULATION
=======================================

$$POI_i := Pd_i \cdot Pc$$
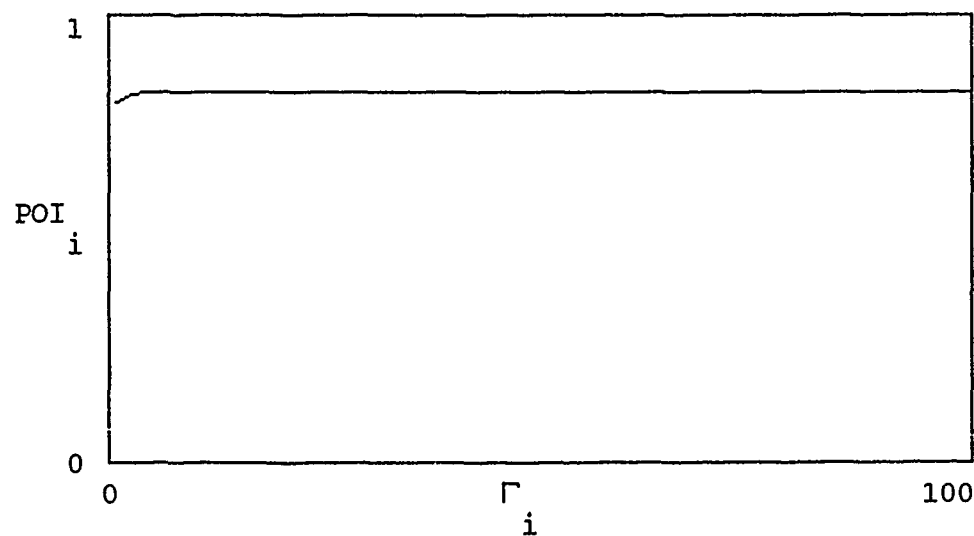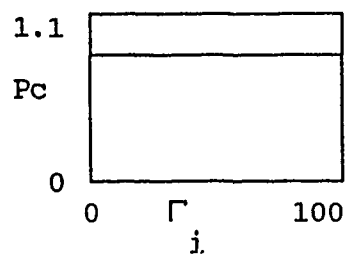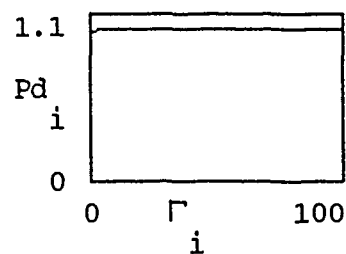
APPENDIX G. MATHCAD WORKSHEET FOR MULTIPLE SYSTEMS (POIM)

PROBABILITY OF INTERCEPT FOR MULTIPLE SYSTEMS
===

POIM - This file calculates the probability of intercept for multiple interceptors operations against a moving or fixed emitter, using multiple stationary scanning superhet receivers and a common scaning antenna

$$data := READPRN \begin{bmatrix} setup \\ \quad\quad prn \end{bmatrix}$$

Parameter definition
=====================

$PtGt := data_0$

$diam := data_1$

$freq := data_2$

$dist := data_3$

$ht := data_4$

$hr := data_5$

$Br := data_6$

$FndB := data_7$

$Ds := data_8$

$Pfa := data_9$

$ton := data_{10}$

$toff := data_{11}$

$ts := data_{12}$

$Ts := data_{13}$

$ta := data_{14}$

$Ta := data_{15}$

$K := data_{16}$

Received signal power calculation
================================

$PtGtdBm := 30 + 10 \cdot \log(PtGt)$

$$\lambda := \frac{3 \cdot 10^{8}}{freq}$$

$$Gr := 0.54 \cdot \left[\frac{\pi \cdot diam}{\lambda}\right]^{2}$$

$GrdB := 10 \cdot \log(Gr)$

$$Lp_{fs} := 32.45 + 20 \cdot \log\left[\frac{freq}{10^{6}}\right] + 20 \cdot \log(dist)$$

$$Lp_{sp} := 108 + 20 \cdot \log\left[\frac{freq}{10^{6}}\right] + 40 \cdot \log(dist) - 20 \cdot \log(ht \cdot hr) + 12$$

$$g2 := 2 \cdot \left| \sin\left[2 \cdot \pi \cdot \left[\frac{ht \cdot hr}{dist \cdot \lambda}\right]\right] \right|$$

$$Si_{fs} := PtGtdBm + GrdB + 10 \cdot \log(g2) - Lp_{fs}$$

$$Si_{sp} := PtGtdBm + GrdB + 10 \cdot \log(g2) - Lp_{sp}$$

Noise power calculation
========================

$$NidBm := -114 + 10 \cdot \log\left[\frac{Br}{10^{6}}\right]$$

Sweeping loss calculation
==========================

$$LsdB := \left[1 + 0.195 \cdot \left[\frac{Ds}{Ts \cdot Br^{2}}\right]^{2}\right]^{-0.25}$$

Signal to noise ratio calculation

$$SNRdB_{fs} := Si_{fs} - NidBm - LsdB - FndB$$

$$SNR_{fs} := 10^{\dfrac{SNRdB_{fs}}{10}}$$

$$SNRdB_{sp} := Si_{sp} - NidBm - LsdB - FndB$$

$$SNR_{sp} := 10^{\dfrac{SNRdB_{sp}}{10}}$$

Radio horizon calculation

$$RHZ := 4.12 \cdot \left[\sqrt{ht} + \sqrt{hr} \right]$$

$$SNR := \Phi(RHZ - dist) \cdot SNR_{fs} + \Phi(dist - RHZ) \cdot SNR_{sp}$$

PROBABILITY OF DETECTION CALCULATION

$$A := \sqrt{\ln\left[\dfrac{1}{Pfa}\right]} \qquad\qquad B := \sqrt{SNR}$$

$$Pd := \dfrac{1}{2} \cdot (1 - erf(A - B)) \quad \ldots$$

$$+ \dfrac{\exp\left[-(A - B)^2\right]}{4 \cdot \sqrt{\pi} \cdot B} \cdot \left[0.75 - \dfrac{A}{4 \cdot B} + \dfrac{1 + 2 \cdot (A - B)^2}{16 \cdot B^2} \right]$$

Probability of coincidence calculation

$$\Gamma := K \cdot \frac{ta \cdot ts}{Ta \cdot Ts}$$

$$Pon := \frac{ton}{ton + toff}$$

$$k := 1 \,..500$$

$$t_k := k$$

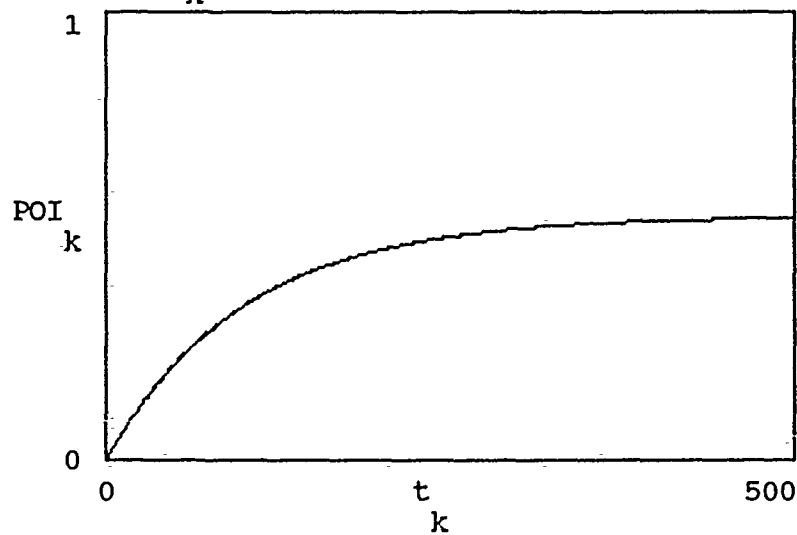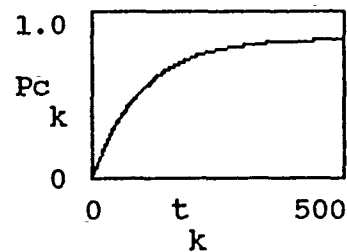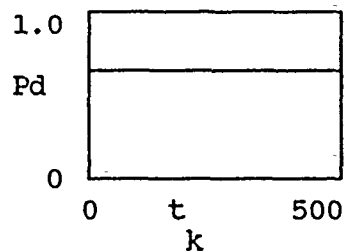$$Pob_k := 1 - \exp\left[-\Gamma \cdot t_k\right]$$

$$Pc_k := Pon \cdot Pob_k$$

PROBABILITY OF INTERCEPT CALCULATION

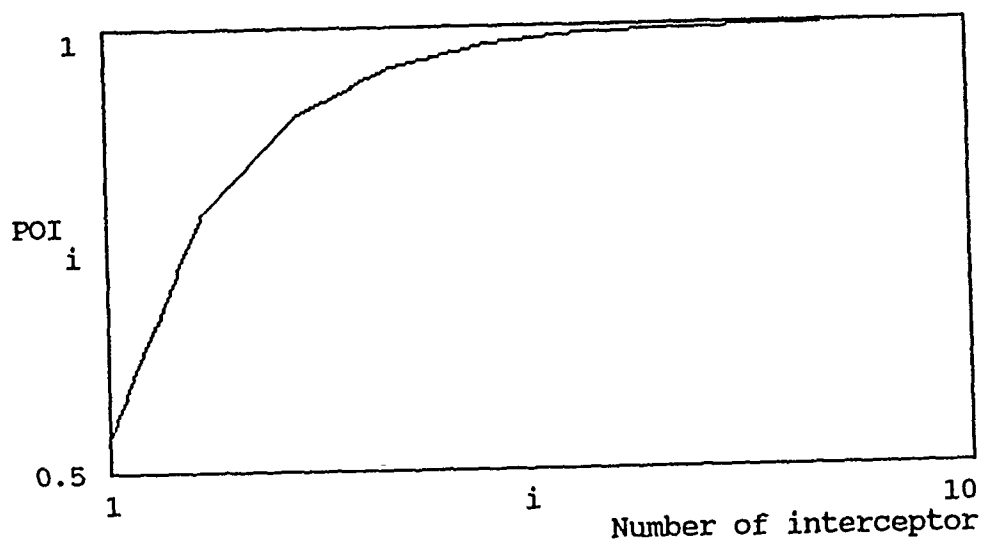$$POI_k := Pd \cdot Pc_k$$

PROBABILITY OF INTERCEPT FOR MULTIPLE SYSTEMS

$POI_0 := POI_{500}$ Define initial POI as POI(t=500) for one system

$i := 1 .. 10$

                       POI for 1 system

$POI_i := 1 - \left[1 - POI_0\right]^i$ $POI_1 = 0.538$

POI_i vs. Number of interceptor (y-axis from 0.5 to 1, x-axis from 1 to 10)

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